

## 1. Applications of the first derivative.

- a) Find the interval(s) where the function is increasing and the interval(s) where the function is decreasing.

$$f(x) = x^3 - 3x + 4$$

$$g(x) = \frac{1}{2x+3}$$

$$h(x) = (x-5)^{\frac{2}{3}}$$

$$r(x) = x\sqrt{x+1}$$

- b) Find any relative maxima and relative minima of the function.

$$f(x) = x^3 - 3x + 6$$

$$g(x) = \frac{x}{x+1}$$

$$h(x) = x\sqrt{x-4}$$

## 2. Applications of the second derivative.

- a) Find any inflection points of the function, the interval(s) on which it is concave upward, and the interval(s) on which it is concave downward.

$$f(x) = x^3 - x$$

$$g(x) = \frac{x}{1-x}$$

$$h(x) = 2 + \frac{3}{x}$$

- b) Determine if the given critical point(s) is a maximum or minimum using the given second derivative.

$$x = 0, x = 1; f^{(2)}(x) = 2x - 1$$

$$x = 0; g^{(2)}(x) = 3x^2 + 1$$

$$x = 0; h^{(2)}(x) = -\frac{6(4x-3)}{(2x+3)^4}$$

- c) Sketch the graph of a function with the following properties:  $f(0) = 0$ ,  $f'(0)$  is undefined, and  $f''(x) < 0$  if  $x \neq 0$ .

## 3. Find all asymptotes of the given function.

a)  $f(x) = \frac{x+1}{2x-1}$

b)  $g(x) = \frac{2x}{x^2+x-2}$

c)  $h(x) = 1 + \frac{2}{x-3}$

d)  $r(x) = \frac{1-x^2}{x^2+x}$

## 4. Find the absolute maximum and absolute minimum of the function over the given domain (when such extrema exist).

a)  $f(x) = x^{\frac{2}{3}}$  over  $\mathbb{R}$

b)  $g(x) = \frac{x}{1-x}$  over  $[2, 6]$

c)  $h(x) = x^3 - 3x^2 - 1$  over  $[0, 3]$

d)  $f(x) = \frac{x}{x^2+1}$  over  $\mathbb{R}$ .

5. Optimization.

- a) A stone thrown straight up from the roof of an 80-ft building has a height after  $t$  seconds of  $h(t) = -16t^2 + 64t + 80$  ft. What is the maximum height reached by the stone?
- b) The demand equation for a product is  $p = -0.00042x + 6$  for  $0 \leq x \leq 12,000$ . The cost function for the same product is  $C(x) = 600 + 2x - 0.00002x^2$  for  $0 \leq x \leq 20,000$ . How many should the company produce in order to maximize revenue?
- c) If an open box has a square base and a volume of  $108 \text{ in}^3$ , find the dimensions that minimize the amount of material in the box.
- d) Find the radius and height of a right circular cylinder with closed ends and a volume of  $36 \text{ in}^3$  that has a minimum surface area.
- e) Suppose the cost incurred for operating a cruise ship for one hour at  $v$  mph is  $a + bv^3$  dollars, where  $a$  and  $b$  are positive constants. What speed will minimize operating costs?
- f) A book is to have pages with a printed area of  $54 \text{ in}^2$  and margins of 0.5 in on the top, bottom, and right sides, and a margin of 1 in on the left. What dimensions will minimize the amount of paper used per page?

4. Exponential functions.

- a) Simplify the expression:

$$(2x^3)(-4x^{-2})$$

$$\frac{(a^m a^{-n})^{-2}}{(a^{m+n})^2}$$

- b) Solve the equation for  $x$ :

$$5^{-x} = 5^3$$

$$4^{2x-1} = 4^{x+3}$$

$$7^{x-x^2} = \frac{1}{49^x}$$