

INFORMATION: Calculators are allowed but no other materials are permitted. Always show your work: problems with little or no work shown may receive little or no credit. You may find the quadratic formula useful:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1. Find the interval(s) on which the function $f(x) = \frac{x}{x^2+1}$ is increasing and the interval(s) on which it is decreasing.

2. Find all critical points of the function $f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2$ and determine if each is a relative maximum, relative minimum, or neither.

3. Is the function $f(x) = \frac{1}{5}x^5 - 3x^4 + 6x$ concave up or concave down over the interval $(0, 9)$?

4. A function f has a critical point at $x = 2$ and its second derivative is $f''(x) = \frac{6-2x}{x^4}$. Determine if f has a relative maximum or a relative minimum at the critical point.

5. Find all asymptotes of the function $f(x) = \frac{x}{3-x}$.

6. Find all asymptotes of the function $g(x) = \frac{x^2-100}{x^2+x-90}$.

7. Find the absolute maximum and absolute minimum of the function $f(x) = \frac{1}{1+3x^2-x^3}$ over the domain $[0, 3]$. It may be useful to know that $1 + 3x^2 - x^3 \neq 0$ when $0 \leq x \leq 3$.

8. The speed of a cheetah (kph) t seconds into its pursuit of a gazelle is given by $f(t) = \frac{1}{3}t^3 - 6t^2 + 32t$ for $0 \leq t \leq 10$. What are the maximum and minimum speeds attained by the cheetah over the 10 seconds of pursuit?

9. By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, an open box may be made. If the cardboard is 8 ft. long and 5 ft. wide, find the dimensions that will maximize the volume of the box.

10. Solve the equation $4^{2x-1} = 64$.