1.

$$\int_0^1 x^2(2x^3-1)^4 dx$$

First find an antiderivative by solving $\int x^2(2x^3-1)^4 dx$. Let $u = 2x^3 - 1$, giving $\frac{1}{6}du = x^2 dx$. Then

$$\int x^2(2x^3-1)^4 dx = \int \frac{1}{6}u^4 du = \frac{u^5}{30} + C = \frac{(2x^3-1)^5}{30} + C$$

Choose a value for C and apply the FTC to find the value of the definite integral.

$$\int_0^1 x^2(2x^3-1)^4 dx = \left.\frac{(2x^3-1)^5}{30}\right|_0^1 = \frac{1}{15}$$

2.

$$\int_1^3 x\sqrt{3x^2-2}dx$$

Find an antiderivative by solving $\int x\sqrt{3x^2-2}dx$. Let $u = 3x^2 - 2$, giving $\frac{1}{6}du = xdx$. Then

$$\begin{aligned}\int x\sqrt{3x^2-2}dx &= \int \frac{1}{6}u^{\frac{1}{2}} du \\ &= \frac{1}{6}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C \\ &= \frac{1}{9}(3x^2-2)^{\frac{3}{2}} + C\end{aligned}$$

Choose a value for C and apply the FTC to find the value of the definite integral.

$$\int_1^3 x\sqrt{3x^2-2}dx = \left.\frac{1}{9}(3x^2-2)^{\frac{3}{2}}\right|_1^3 = \frac{124}{9}$$

3. Find the average value of $f(x) = (2x-1)^{\frac{5}{2}}$ over $[1, 5]$.

The average value of f over $[1, 5]$ is

$$\frac{1}{4}\int_1^5 (2x-1)^{\frac{5}{2}} dx.$$

Using substitution method 2 with $u = 2x - 1$ and $\frac{1}{2}du = dx$ we have

$$\begin{aligned}\frac{1}{4}\int_1^5 (2x-1)^{\frac{5}{2}} dx &= \frac{1}{8}\int_1^9 u^{\frac{5}{2}} du \\ &= \left.\frac{1}{8}\left(\frac{2}{7}\right)u^{\frac{7}{2}}\right|_1^9 \\ &= \frac{1}{28}\left(9^{\frac{7}{2}} - 1\right) \\ &= \frac{1093}{14}\end{aligned}$$

4.

$$\int_0^1 x\sqrt{x+1}dx$$

Let $u = x + 1$ and $du = dx$. Solving for x gives $x = u - 1$. Use substitution method two and change the limits of integration:

$$\begin{aligned}\int_0^1 x\sqrt{x+1}dx &= \int_1^2 (u-1)\sqrt{u}du \\ &= \int_1^2 u^{\frac{3}{2}} - u^{\frac{1}{2}} du \\ &= \left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_1^2 \\ &\approx 0.6438\end{aligned}$$

5. Find the average value of $f(x) = e^{1-x}$ over $[-1, 4]$.

The average value of f over $[-1, 4]$ is

$$\begin{aligned}\frac{1}{5}\int_{-1}^4 e^{1-x}dx &= -\frac{1}{5}e^{1-x}\Big|_{-1}^4 \\ &= -\frac{1}{5}\left(e^{-3} - e^2\right) \\ &\approx 1.4679\end{aligned}$$