Ignoring the effects of friction, an object falling from a height of 64 feet under the influence of gravity will be $p(t) = 64 - 16t^2$ feet above the ground t seconds after it begins falling. Our goal today is to figure out how fast the object is moving when it hits the ground.

1. Determine the time at which the object hits the ground (i.e. reaches height 0). Call this time t_0.

2. Find the *average velocity* of the object by dividing the total change in height (-64, a negative number because it is falling) by the total duration of the fall (t_0).

3. The average velocity of the object over the last a seconds of its fall is

$$\frac{p(t_0 - a) - p(t_0)}{-a}.$$

Use this to find the average velocity over the last $a = 1, \frac{1}{2}$, and $\frac{1}{4}$ seconds.

4. The velocity of the object when it hits the ground is exactly

$$\lim_{a \to 0} \left[\frac{p(t_0 + a) - p(t_0)}{a} \right].$$

Find this velocity.

5. The velocity of the object after one second of its fall is

$$\lim_{a \to 0} \left[\frac{p(1+a) - p(1)}{a} \right].$$

Find this velocity.

6. Sketch a graph of the function p as carefully as you can. Then sketch as carefully as you can the graph of the line passing through the point $(1, p(1))$ with slope equal to your answer for #5. This line is called the *tangent line* for the curve $y = p(t)$ at the point $(1, p(1))$.