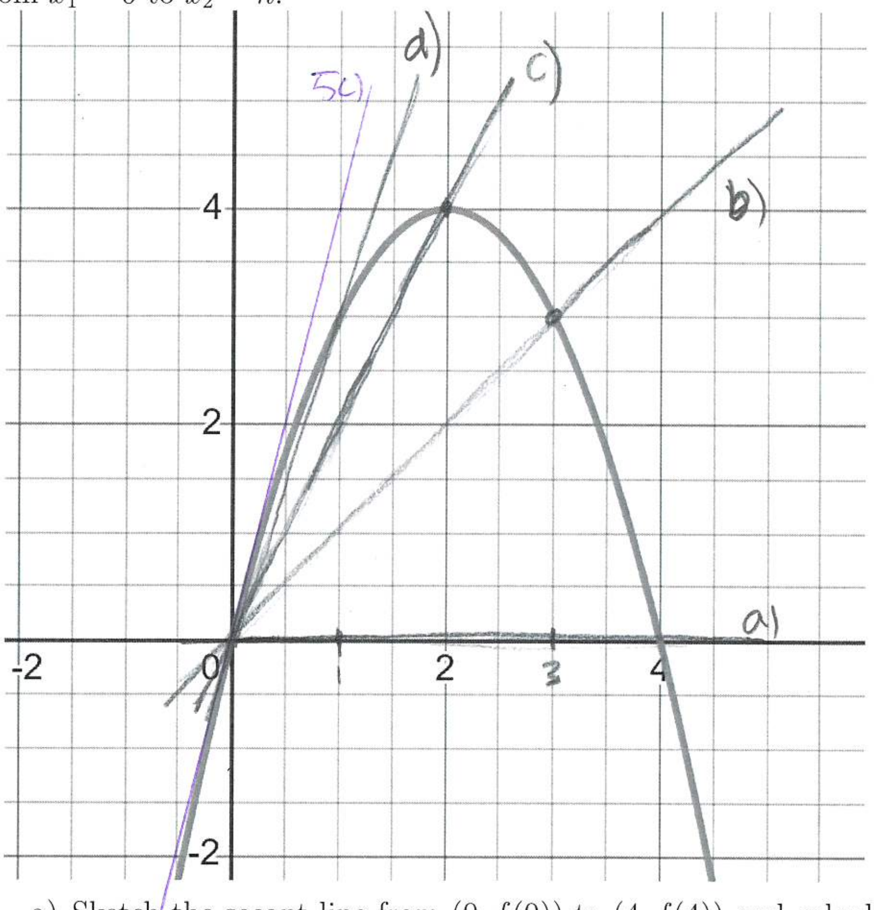
SLOPES AND RATES OF CHANGE

Definition (see p. 129 of the textbook). The **average rate of change** of $f(x)$ as x changes from x_1 to x_2 (when $x_1 \neq x_2$) is

$$\boxed{\frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

This is the same as the **slope of the secant line** thorugh the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

1. Shown below is a graph of the function $f(x) = 4x - x^2$. We wil consider the average rate of change from $x_1 = 0$ to $x_2 = h$.



x	$f(x)$
0	0
1	3
2	4
3	3
4	0

a) Sketch the secant line from $(0, f(0))$ to $(4, f(4))$ and calculate its slope.

$$\frac{f(4)-f(0)}{4-0} = \frac{0-0}{4-0} = 0$$

b) Sketch the secant line from $(0, f(0))$ to $(3, f(3))$ and calculate its slope.

$$\frac{f(3)-f(0)}{3-0} = \frac{3-0}{3-0} = 1$$

c) Sketch the secant line from $(0, f(0))$ to $(2, f(2))$ and calculate its slope.

$$\frac{f(2)-f(0)}{2-0} = \frac{4-0}{2-0} = 2$$

d) Sketch the secant line from $(0, f(0))$ to $(1, f(1))$ and calculate its slope.

$$\frac{f(1)-f(0)}{1-0} = \frac{3-0}{1-0} = 3$$

Date: September 12, 2022.

Definition (see p. 139 of the textbook). The **instantaneous rate of change** (also called the **derivative**) of $f(x)$ at $x = a$ is

$$\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

This is the same as the **slope of the tangent line** at the point $(a, f(a))$.

2. Continue working with the function $f(x) = 4x - x^2$. Our goal now is to find the instantaneous rate of change of $f(x)$ at $x = 0$.

a) Does your work for problem 1 suggest an answer? Make a guess if you can.

The next number in the sequence of slopes $(0, 1, 2, 3)$ is 4.

b) Calculate the limit $\lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$.

$$f(0+h) = 4(0+h) - (0+h)^2 = 4h - h^2$$

$$\lim_{h \to 0} \frac{4h - h^2 - 0}{h} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}}(4-h) = 4.$$

c) Find an equation for the tangent line (use the point $(0, 0)$ and the slope you just calculated). Add this line to the graph on page 1.

$$y - 0 = 4(x - 0) \qquad y = 4x$$

Definition. The **difference quotient** of $f(x)$ is

$$\boxed{\frac{f(x+h) - f(x)}{h}}$$

Note that the derivative of f at x is the limit of the difference quotient as $h \to 0$.

3. Calculate the difference quotient for the following functions and reduce the fraction.

a) $f(x) = 4x$

$$\frac{4(x+h) - 4x}{h} = \frac{\cancel{4x} + 4h - \cancel{4x}}{h} = \frac{4h}{h} = 4$$

b) $f(x) = x^2$

$$\frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \frac{\cancel{h}}{\cancel{h}}(2x+h)$$

$$= 2x + h$$

c) $f(x) = 4x - x^2$

see a) see b)

$$\frac{4(x+h) - (x+h)^2 - [4x - x^2]}{h} = \frac{4(x+h) - 4x}{h} - \left[\frac{(x+h)^2 - x^2}{h}\right]$$

$$= 4 - 2x + h$$

And the limit as $h \to 0$: $4 - 2x$ ← the derivative of $f(x) = 4x - x^2$

[illegible]