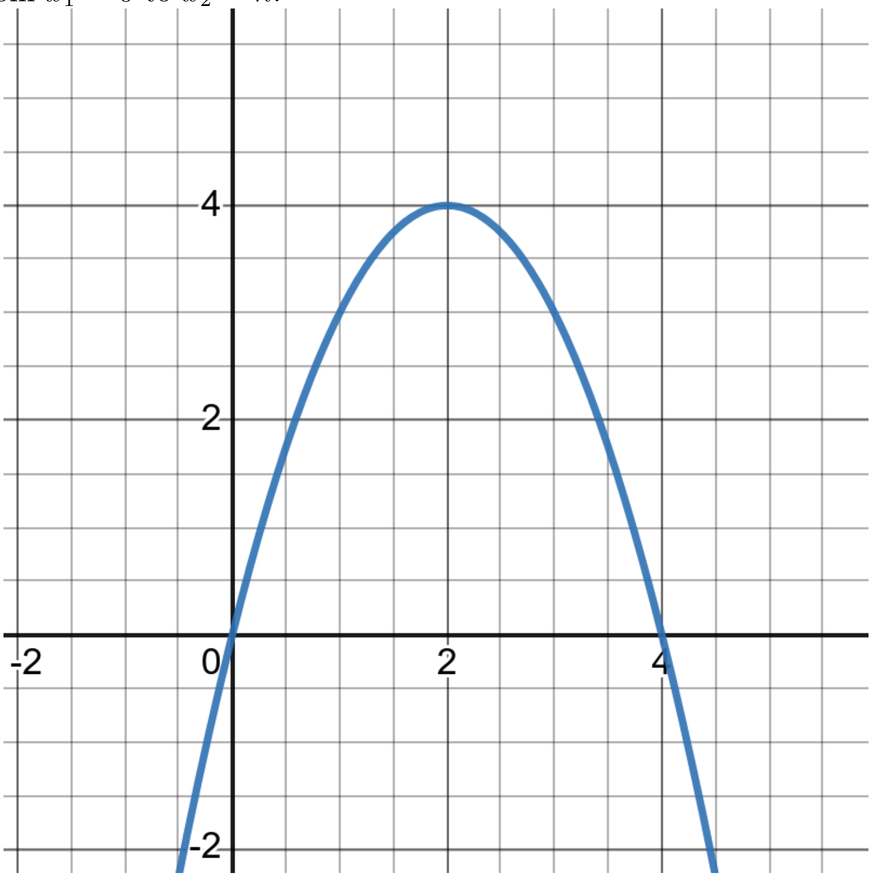
# SLOPES AND RATES OF CHANGE

**Definition** (see p. 129 of the textbook)**.** The **average rate of change** of $f(x)$ as $x$ changes from $x_1$ to $x_2$ (when $x_1 \neq x_2$) is

$$\boxed{\frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

This is the same as the **slope of the secant line** thorugh the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

**1.** Shown below is a graph of the function $f(x) = 4x - x^2$. We wil consider the average rate of change from $x_1 = 0$ to $x_2 = h$.




a) Sketch the secant line from $(0, f(0))$ to $(4, f(4))$ and calculate its slope.

b) Sketch the secant line from $(0, f(0))$ to $(3, f(3))$ and calculate its slope.

c) Sketch the secant line from $(0, f(0))$ to $(2, f(2))$ and calculate its slope.

d) Sketch the secant line from $(0, f(0))$ to $(1, f(1))$ and calculate its slope.

*Date*: September 12, 2022.

**Definition** (see p. 139 of the textbook)**.** The **instantaneous rate of change** (also called the **derivative**) of $f(x)$ at $x = a$ is

$$\boxed{\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}}$$

This is the same as the **slope of the tangent line** at the point $(a, f(a))$.

**2.** Continue working with the function $f(x) = 4x - x^2$. Our goal now is to find the instantaneous rate of change of $f(x)$ at $x = 0$.

a) Does your work for problem 1 suggest an answer? Make a guess if you can.

b) Calculate the limit $\lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h}$.

c) Find an equation for the tangent line (use the point $(0, 0)$ and the slope you just calculated). Add this line to the graph on page 1.

**Definition.** The **difference quotient** of $f(x)$ is

$$\boxed{\frac{f(x+h) - f(x)}{h}}$$

Note that the derivative of $f$ at $x$ is the limit of the difference quotient as $h \to 0$.

**3.** Calculate the difference quotient for the following functions and reduce the fraction.

a) $f(x) = 4x$

b) $f(x) = x^2$

c) $f(x) = 4x - x^2$