HIGHER-ORDER DERIVATIVES

1. On the first worksheet we dealt with a mathematical model for the vertical position of a model rocket. The altitude (in meters) t seconds after launch is given by

$$h(t) = \begin{cases} 40t^2 & \text{if } t \leq 2 \\ 160 + 160(t-2) - 4(t-2)^2 & \text{if } t > 2 \end{cases}$$

This is a piecewise function because the rocket engine stops 2 seconds into the flight, after which the rocket moves only under the influences of gravity and friction.

a) Calculate the derivative $h'(t)$ and interpret what you find. Does the derivative exist at $t = 2$?

b) Calculate the second derivative $h''(t)$ and interpret what you find. Does the second derivative exist at $t = 2$?

c) Calculate the third derivative $h^{(3)}(t)$ and interpret what you find.

Date: September 28, 2022.

2. Real mathematical models for the absorption and metabolism of a drug are too complex for us right now, but we can work with a simplified example. Suppose the concentration of a intravenously inject drug (in mg/l) t minutes after injection is given by

$$C(t) = \frac{10t}{t^2 + 1}$$

a) Calculate the derivative $C'(t)$ and interpret what you find. What are the units of $C'(t)$? For which values of t is $C'(t)$ positive? For which values is it negative?

b) Use Desmos to plot the graph $y = C(t)$. Compare the features of the graph with your answers for part a.

c) Calculate the second derivative $C''(t)$. What are the units of $C''(t)$?

Challenge. Continue working with $C(t)$ from the last problem. Simplify $C''(t)$ and find the values of t for which $C''(t)$ is positive and the values for which it is negative. Compare with the graph of $y = C(t)$.