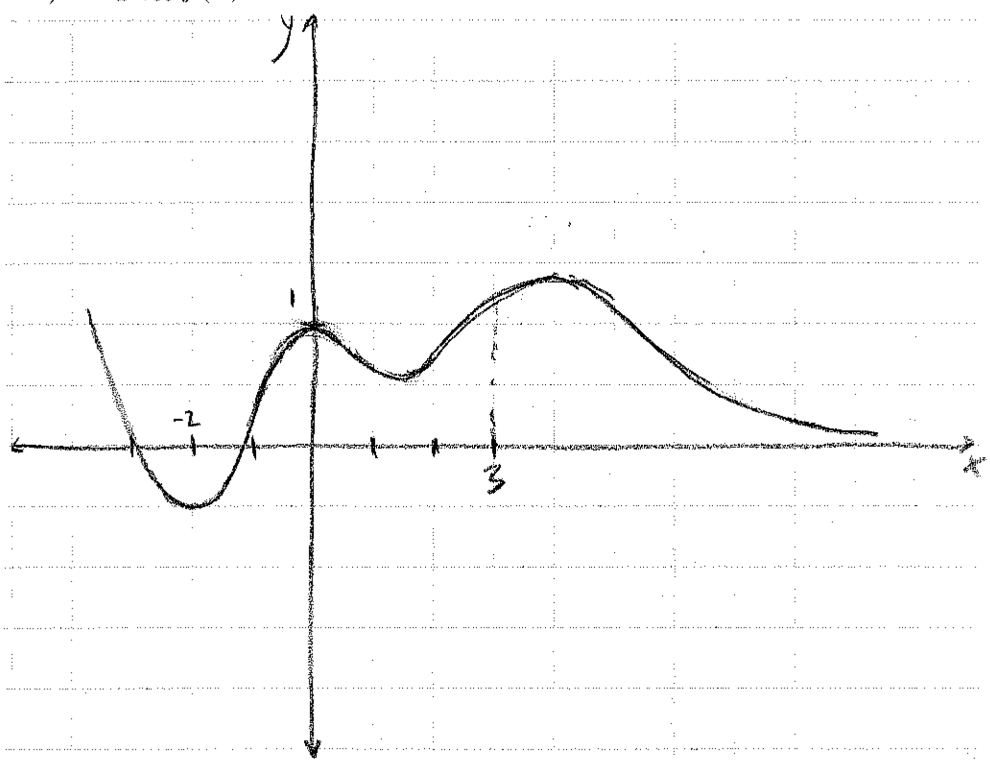
The Shapes of Graphs and Absolute Extremes

~~FIRST DERIVATIVES AND OPTIMIZATION~~

1. Sketch the graph of a function $y = f(x)$ such that all of the following are true:

a) $f'(-2) = 0$ and $f''(-2) > 0$
b) $f(0) = 1$, $f'(0) = 0$, and $f''(0) < 0$
c) $f'(3) > 0$ and $f''(3) < 0$
d) $\lim_{x\to\infty} f(x) = 0$



Many possibilities. Must have a local min at $x=-2$ and a local max of 1 at $x=0$. Can't have a local extreme at $x=3$.

2. Why does $f(x) = x - \frac{1}{x}$ not have an absolute minimum on the interval $(0, 2]$? Does it have an absolute maximum?

$\lim_{x\to 0^+} x - \frac{1}{x} = -\infty$, so there can't be an abs. min on $(0,2]$.

$f'(x) = 1 + \frac{1}{x^2}$. Note that $f'(x) > 1$ for all x. This means f only ever increases. The abs. max over $(0,2]$ must happen at $x=2$. The max is $f(2) = 2 - \frac{1}{2} = \frac{3}{2}$.

3. Find the absolute minimum and maximum of $f(x) = x - 3x^{2/3}$ over the interval $[-1, 10]$.

$f'(x) = 1 - 2x^{-1/3}$ undefined @ $x = 0$.

$\left(0 = 1 - 2x^{-1/3}\right) x^{1/3}$

$0 = x^{1/3} - 2$

$x = 8$

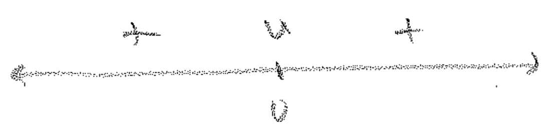
Critical values: $x = 0, 8$

x	$f(x) = x - 3x^{2/3}$
-1	$-1 - 3(-1)^{2/3} = -4$
0	$0 - 3(0)^{2/3} = 0$
8	$8 - 3(8)^{2/3} = -4$
10	$10 - 3(10)^{2/3} \approx -3.9$

Abs. min of -4 at $x = -1$ and $x = 8$.

Abs. max of 0 at $x = 0$.

$f''(x) = \frac{2}{3} x^{-4/3}$



$0 = f(x) = x - 3x^{2/3}$ @ $x = 0$ and $x = 27$

$0 = x^{2/3}(x^{1/3} - 3)$

$\downarrow \qquad \downarrow$

$x = 0 \qquad x = 27$

$f'(x) = 1 - \frac{2}{x^{1/3}}$