OPTIMIZATION

1. An intravenous drug is administered to a patient. The concentration of the drug in the bloodstream, in nanograms per milliliter (ng/mL), t minutes after injection is given by $C(t) = 110te^{-0.006t}$. What is the peak concentration of the drug and when does this occur?

Domain: $[0, \infty)$.

$C(t) = 110te^{-0.006t}$

$C'(t) = 110e^{-0.006t} + 110t(-0.006)e^{-0.006t}$

$= 110e^{-0.006t}\left[1 - 0.006t\right]$ ← defined everywhere

$0 = C'(t)$ when $1 - 0.006t = 0$: $t = \frac{1}{0.006} = \frac{500}{3} \approx 166.67$ min

$$\begin{array}{ccccc} 110 & + & 0 & - & C'(t) \\ \hline 0 & & \frac{500}{3} & & \end{array}$$

$C(t)$ increases until $\frac{500}{3}$ min, then decreases. Therefore the absolute maximum concentration is $C\left(\frac{500}{3}\right) = 110\left(\frac{500}{3}\right)e^{-0.006\left(\frac{500}{3}\right)}$

$= \frac{55000}{3e}$

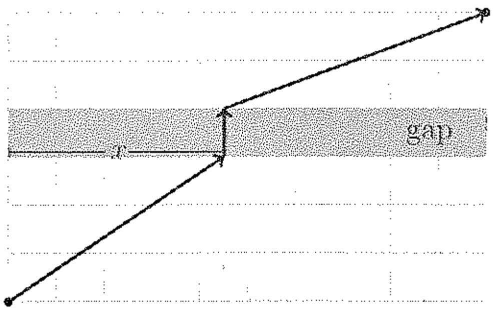
≈ 6744.46 ng/mL.

This occurs after $\frac{500}{3}$ minutes; that's 2 hr 46 min 40 sec.

Date: November 4, 2022.

2. Ants tend to find the shortest path between their home and a food source, but that's generally a straight line. Can they also find the the shortest path when it isn't a straight line? To investigate this, I propose the following: place food on a separate surface from the ants and see if they still find the shortest path. The ants should be able to bridge the gap between the surfaces, but only by going straight across (no diagonal crossings). To determine if the ants will be able to find the shortest path to the food, we first need to find that path. The diagram below shows the experimental setup with grid lines every 1 inch; the ants start at the bottom left and the food is at the top right. The ants cross the gap x inches from the left side (remember that they must go straight across, not diagonally). Use the following steps to find the minimum distance from the ants to the food.

a) Express the distance from the ants to the food as a function of x.
b) What closed interval of values for x do we care about?
c) Verify that your function is continuous on the interval you just found, then find the absolute minimum for the function over the interval.



a) $f(x) = \sqrt{9+x^2} + 1 + \sqrt{4+(10-x)^2}$

b) $[0, 10]$

c) f is continuous everywhere.

$$f'(x) = \frac{x}{\sqrt{9+x^2}} - \frac{(10-x)}{\sqrt{4+(10-x)^2}}$$ ← defined everywhere.

$$0 = \frac{x}{\sqrt{9+x^2}} - \frac{10-x}{\sqrt{4+(10-x)^2}}$$

$$0 = \frac{x^2}{9+x^2} - \frac{(10-x)^2}{4+(10-x)^2}$$

$$0 = [4+(10-x)^2]x^2 - (10-x)^2(9+x^2) = 4x^2 - 9(10-x)^2 = -5x^2 + 180x - 900$$

$$0 = x^2 - 36x + 180$$

$$x = \frac{36 \pm \sqrt{(36)^2 - 4(180)}}{2} = \frac{36 \pm 24}{2} = \boxed{6} \text{ or } 30$$

↳ the only critical value in the domain

$f(0) = 3 + 1 + \sqrt{4+100} = 4 + \sqrt{104} \approx 14.20$ in

$f(6) = \sqrt{9+36} + 1 + \sqrt{4+16} = \sqrt{45} + 1 + \sqrt{20} \approx \boxed{12.18 \text{ in}}$ ← min

$f(10) = \sqrt{109} + 3 \approx 13.44$ in