

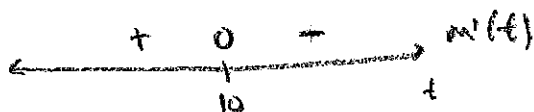
APPLICATIONS OF THE DEFINITE INTEGRAL

1. The rate of memorizing information increases over time to a maximum, then decreases. Suppose this information memorization rate is modeled by $m(t) = -0.009t^2 + 0.18t$ words per minute (time t is measured in minutes).

- When is the memorization rate at a maximum?
- Make an educated guess about whether more words are memorized from time $t_1 = 0$ to time $t_2 = 10$ or from time $t_1 = 5$ to time $t_2 = 15$.
- The number of words memorized from time t_1 to time t_2 is $\int_{t_1}^{t_2} m(t) dt$. Use this to find exact answers for the number of words memorized over the two intervals in the previous part. Were you right?

a) $m'(t) = -0.018t + 0.18$ ← defined everywhere

$$0 = -0.018t + 0.18 \Rightarrow t = 10$$



Max memorization rate occurs at $t = 10$ min.

b) Since $[5, 15]$ includes 10 right in the middle, I think this will be when more words are memorized.

$$\begin{aligned} \text{c) } \int_0^{10} -0.009t^2 + 0.18t dt &= -0.003t^3 + 0.09t^2 \Big|_0^{10} \\ &= -3 + 9 - 0 \\ &= \boxed{6} \end{aligned}$$

$$\begin{aligned} \int_5^{15} -0.009t^2 + 0.18t dt &= -0.003t^3 + 0.09t^2 \Big|_5^{15} \\ &= -0.003(15^3) + 0.09(15^2) - [-0.003(5^3) + 0.09(5^2)] \\ &= 0.003(5^3 - 15^3) + 0.09(15^2 - 5^2) \\ &= -9.75 + 18 = \boxed{8.25} \end{aligned}$$

(continued on the reverse)

2. Solve the initial value problems. It may help to know that there are 5280 feet per mile.

- a) A car accelerates at a constant rate from 0 mph to 60 mph in 30 sec. How far did it travel?
 b) A car decelerates at a constant rate from 60 mph to 0 mph in 5 sec. How far did it travel?

a) Acceleration: $60 \text{ mi/hr} / 30 \text{ sec} = \frac{60 \text{ mi/hr}}{(1/120) \text{ hr}} = 60 \cdot 120 \frac{\text{mi}}{\text{hr}^2}$

Velocity: $v(0) = 0$ and $v'(t) = 60 \cdot 120$

$$v(t) = \int 60 \cdot 120 \, dt = 60 \cdot 120 t + C$$

$0 = v(0) = C$ so $v(t) = 60 \cdot 120 t$

Distance traveled: $\int_0^{1/120} |60 \cdot 120 t| \, dt = 30 \cdot 120 t^2 \Big|_0^{1/120}$
 $= 30 \cdot 120 \left(\frac{1}{120} \right)^2 - 0 = \frac{30}{120} = \frac{1}{4} \text{ mi}$
everything is positive, so we ignore the abs. value
 $\frac{1}{120}$ → be consistent with units: $\frac{1}{120} \text{ hr} = 30 \text{ sec}$

b) Acceleration: $\frac{-60 \text{ mi/hr}}{5 \text{ sec}} = \frac{-60 \text{ mi/hr}}{(1/12 \cdot 60) \text{ hr}} = -60 \cdot 720 \frac{\text{mi}}{\text{hr}^2}$

velocity: $v(0) = 60$ and $v'(t) = -60 \cdot 720 \frac{\text{mi}}{\text{hr}^2}$

$$v(t) = \int -60 \cdot 720 \, dt = -60 \cdot 720 t + C$$

$60 = v(0) = C$ so $v(t) = -60 \cdot 720 t + 60$

Distance traveled: $\int_0^{1/720} |-60 \cdot 720 t + 60| \, dt = -30 \cdot 720 t^2 + 60 t \Big|_0^{1/720}$
 $= -30 \cdot 720 \left(\frac{1}{720} \right)^2 + 60 \left(\frac{1}{720} \right) - 0$
 $= \frac{30}{720} = \frac{1}{24} \text{ mi} = 220 \text{ ft}$
again, the abs. value does nothing