APPLICATIONS OF THE DEFINITE INTEGRAL

1. The rate of memorizing information increases over time to a maximum, then decreases. Suppose this information memorization rate is modeled by $m(t) = -0.009t^2 + 0.18t$ words per minute (time t is measured in minutes).

a) When is the memorization rate at a maximum?

b) Make an educated guess about whether more words are memorized from time $t_1 = 0$ to time $t_2 = 10$ or from time $t_1 = 5$ to time $t_2 = 15$.

c) The number of words memorized from time t_1 to time t_2 is $\int_{t_1}^{t_2} m(t)\, dt$. Use this to find exact answers for the number of words memorized over the two intervals in the previous part. Were you right?

a) $m'(t) = -0.018t + 0.18$ ← defined everywhere

$0 = -0.018t + 0.18 \Rightarrow t = 10$

$$\begin{array}{ccccc} & + & 0 & - & m'(t) \\ \longleftarrow & & \underset{10}{|} & & \longrightarrow t \end{array}$$

Max memorization rate occurs at $t = 10$ min.

b) Since $[5, 15]$ includes 10 right in the middle, I think this will be when more words are memorized.

c) $\int_0^{10} -0.009t^2 + 0.18t\, dt = -0.003t^3 + 0.09t^2 \Big|_0^{10}$

$= -3 + 9 - 0$

$= 6$

$\int_5^{15} -0.009t^2 + 0.18t\, dt = -0.003t^3 + 0.09t^2 \Big|_5^{15}$

$= -0.003(15^3) + 0.09(15^2) - [-0.003(5^3) + 0.09(5^2)]$

$= 0.003(5^3 - 15^3) + 0.09(15^2 - 5^2)$

$= -9.75 + 18 = 8.25$

(continued on the reverse)

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2. Solve the initial value problems. It may help to know that there are 5280 feet per mile.

a) A car accelerates at a constant rate from 0 mph to 60 mph in 30 sec. How far did it travel?

b) A car decelerates at a constant rate from 60 mph to 0 mph in 5 sec. How far did it travel?

a) Acceleration: $60\,\text{mi/hr} \,/\, 30\,\text{sec} = \dfrac{60\,\text{mi/hr}}{(1/120)\,\text{hr}} = 60\cdot 120 \dfrac{\text{mi}}{\text{hr}^2}$

Velocity: $V(0) = 0$ and $V'(t) = 60\cdot 120$

$$v(t) = \int 60\cdot 120\, dt = 60\cdot 120 t + C$$

$$0 = V(0) = C \quad \text{so} \quad v(t) = 60\cdot 120\, t$$

be consistent with units: $\frac{1}{120}\,\text{hr} = 30\,\text{sec}$

Distance traveled: $\displaystyle\int_0^{\frac{1}{120}} |60\cdot 120 t|\, dt = 30\cdot 120 t^2 \Big|_0^{1/120}$

everything is positive, so we ignore the abs. value

$$= 30\cdot 120\left(\frac{1}{120}\right)^2 - 0 = \frac{30}{120} = \frac{1}{4}\,\text{mi}$$

b) Acceleration: $\dfrac{-60\,\text{mi/hr}}{5\,\text{sec}} = \dfrac{-60\,\text{mi/hr}}{1/(12\cdot 60)\,\text{hr}} = -60\cdot 720 \dfrac{\text{mi}}{\text{hr}^2}$

velocity: $v(0) = 60$ and $v'(t) = -60\cdot 720 \frac{\text{mi}}{\text{hr}^2}$

$$V(t) = \int -60\cdot 720\, dt = -60\cdot 720 t + C$$

$$60 = V(0) = C \quad \text{so} \quad V(t) = -60\cdot 720 t + 60$$

again, the abs. value does nothing

Distance traveled: $\displaystyle\int_0^{\frac{1}{720}} |-60\cdot 720 t + 60|\, dt = -30\cdot 720 t^2 + 60t \Big|_0^{1/720}$

$$= -30\cdot 720\left(\frac{1}{720}\right)^2 + 60\left(\frac{1}{720}\right) - 0$$

$$= \frac{30}{720} = \frac{1}{24}\,\text{mi} = 220\,\text{ft}$$