SIR MODELS

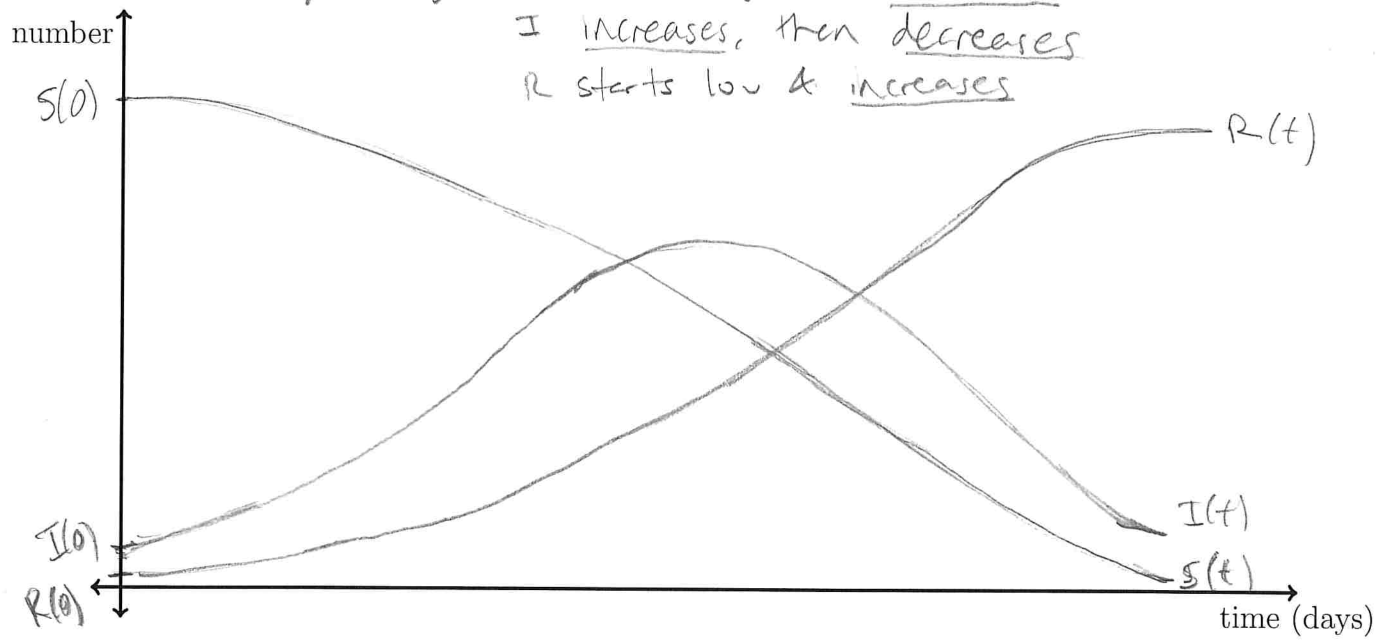
The SIR model divides a population into 3 groups: the susceptible, the infected, and the recovered/resistant. We then track those 3 subpopulations using the following variables:

S is the number of susceptible individuals
I is the number of infected individuals
R is the number of recovered/resistant individuals

All 3 variables change over time, so we'll need to remember that they're functions of time. At all times we assume that the total population is the same: $S(t) + I(t) + R(t)$ is constant.

1. How do you think $S(t)$, $I(t)$, and $R(t)$ should work? Sketch a (rough) graph of all 3 functions on a single set of axes.

Key things: S starts high & decreases
I increases, then decreases
R starts low & increases



2. Now we'll try to be more sophisticated and mathematical. Apply pillar 2 and **track the changes** to fill in the right side of the following equations (we'll do this all together).

$S' = -aSI$
$I' = aSI - bI$
$R' = bI$

a is a positive constant; the communicability* parameter.

Units for a: $\frac{1}{\text{person}\cdot\text{day}}$ "per person day"

- Close to 0 is not very contagious
- larger values mean more rapid spread.

* in this case, a actually combines the communicability of the disease with social factors like the kind and number of contacts between individuals, masking (or lack thereof), and so on.

b is a positive constant: the recovery rate. Units for b: $\frac{1}{\text{day}}$. "per day"

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 with a and b using times measured in days

3. Suppose $S(0) = 100$, $I(0) = 1$, and $R(0) = 0$ and let $\boxed{a = 0.01 \text{ and } b = 0.125.}$ Now we'll go **one step at a time** (pillar 5) to make predictions.

a) Predict $S(1)$, $I(1)$, and $R(1)$

$$S(1) \approx S(0) - aS(0)I(0) = 100 - 0.01(100)(1) = \underline{99}$$

$$I(1) \approx I(0) + aS(0)I(0) - bI(0) = 1 + 0.1(100)(1) - 0.125(1) = \underline{1.875}$$

$$R(1) \approx R(0) + bI(0) = 0 + 0.125(1) = \underline{0.125}$$

could round to nearest integer or think of these as averages.

b) Predict $S(2)$, $I(2)$, and $R(2)$

$$S(2) \approx S(1) - aS(1)I(1) = 99 - 0.01(99)(1.875) = \underline{97.14375}$$

$$I(2) \approx I(1) + aS(1)I(1) - bI(1) = 1.875 + 0.01(99)(1.875) - 0.125(1.875)$$
$$= \underline{3.496875}$$

$$R(2) \approx R(1) + bI(1) = 0.125 + 0.125(1.875) = \underline{0.359375}$$

Use the exact values of the predictions for $S(1)$, $I(1)$, and $R(1)$ for best predictions here. It is also possible to go directly from $S(0)$, $I(0)$, and $R(0)$ to $S(2)$, $I(2)$, and $R(2)$ using:

$$S(2) \approx S(0) - 2aS(0)I(0) = 98$$

$$I(2) \approx I(0) + 2aS(0)I(0) - 2bI(0) = 2.75$$

$$R(2) \approx R(0) + 2bI(0) = 0.25$$

We should expect this to be less accurate: many small steps generally gives a better prediction than few large steps.

c) Predict $S(3)$, $I(3)$, and $R(3)$

$$S(3) \approx S(2) - aS(2)I(2)$$
$$\approx 93.74675449922$$

$$I(3) \approx I(2) + aS(2)I(2) - bI(2)$$

$$\approx 6.4567611328$$

$$R(3) \approx R(2) + bI(2)$$
$$= 0.796484375$$