THE PRODUCT, QUOTIENT, AND CHAIN RULES

Theorem. Building blocks:

i. $\frac{d}{dx}[c] = 0$ for any constant c

ii. $\frac{d}{dx}[x^n] = nx^{n-1}$ $(n \neq 0)$

iii. $\frac{d}{dx}[e^x] = e^x$

iv. $\frac{d}{dx}[\sin x] = \cos x$

v. $\frac{d}{dx}[\cos x] = -\sin x$

Product rule: $\boxed{(fg)' = f'g + g'f}$

Quotient rule: $\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{[g]^2}}$

Chain rule: $\boxed{(f(g(x)))' = f'(g(x))\, g'(x)}$

1. Observe that $\dfrac{1}{x} - 9 = \dfrac{1-9x}{x} = \left(\dfrac{1}{\sqrt{x}} - 3\right)\left(\dfrac{1}{\sqrt{x}} + 3\right)$. Use this to differentiate $f(x) = \frac{1}{x} - 9$ in three different ways.

a) Differentiate directly: $\dfrac{d}{dx}\left[\dfrac{1}{x} - 9\right]$

b) Use the quotient rule: $\dfrac{d}{dx}\left[\dfrac{1-9x}{x}\right]$

c) Use the product rule: $\dfrac{d}{dx}\left[\left(\dfrac{1}{\sqrt{x}} - 3\right)\left(\dfrac{1}{\sqrt{x}} + 3\right)\right]$

d) All three of your solutions should be equal. Are they?

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2. Find an equation for the line tangent to the graph $y = \sqrt{x^2 + 3x}$ at the point $(1, 2)$.

3. Use the chain rule (and any other appropriate rules) to find the following derivatives:

a) $\frac{d}{dx}\left[(x^2 - 1)^5\right]$

b) $\frac{d}{dx}\left[\frac{1}{\sqrt{7x + 3}}\right]$

c) $\frac{d}{dx}\left[e^{x^2}\right]$

d) $\frac{d}{dx}\left[\sin\left(\frac{e^x}{x}\right)\right]$