APPLICATIONS OF THE DEFINITE INTEGRAL

1. The rate of memorizing information increases over time to a maximum, then decreases. Suppose this information memorization rate is modeled by $m(t) = -0.009t^2 + 0.18t$ words per minute (time t is measured in minutes).

a) When is the memorization rate at a maximum?

b) Make an educated guess (use your answer for part a) about whether more words are memorized from time $t_1 = 0$ to time $t_2 = 10$ or from time $t_1 = 5$ to time $t_2 = 15$.

c) The number of words memorized from time t_1 to time t_2 is $\int_{t_1}^{t_2} m(t)\ dt$. Use this to find exact answers for the number of words memorized over the two intervals in the previous part. Were you right?

(continued on the reverse)

Date: November 20, 2023.

2. For this problem, use the fact that $a(t) = v'(t) = s''(t)$. For both parts, assume constant acceleration.

a) A car accelerates from 0 to 100 kph in 10 seconds. How far did it travel over this time?

b) A car decelerates from 100 kph to a stop in 4 seconds. How far did it travel over this time?

3. After WWII ended in 1945, the birth rate in the US increased dramatically (the baby boom). Suppose the birth rate $b(t)$ (in millions of births per year) for 1945 to 1965 was

$$b(t) = 4 + 0.025t$$

where t is the number of years after 1945.

a) According to this model, how many babies were born from 1945 to 1965?

b) The average value of $y = f(x)$ for $a \leq x \leq b$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$. Calculate the average number of births per year over the period 1945 to 1965.

c) Find the accumulation function $B(t)$ for the total number of births in the t years since 1945.

d) Starting after the war, how long did it take for 20 million births?