Solutions

APPLICATIONS OF THE DEFINITE INTEGRAL

1. The rate of memorizing information increases over time to a maximum, then decreases. Suppose this information memorization rate is modeled by $m(t) = -0.009t^2 + 0.18t$ words per minute (time t is measured in minutes).

a) When is the memorization rate at a maximum?

b) Make an educated guess (use your answer for part a) about whether more words are memorized from time $t_1 = 0$ to time $t_2 = 10$ or from time $t_1 = 5$ to time $t_2 = 15$.

c) The number of words memorized from time t_1 to time t_2 is $\int_{t_1}^{t_2} m(t)\, dt$. Use this to find exact answers for the number of words memorized over the two intervals in the previous part. Were you right?

a) $m'(t) = -0.018t + 0.18$

$0 = m'(t) = -0.018t + 0.18 \implies t = \frac{0.18}{0.018} = 10$ minutes.

b) I think more will be memorized from $t_1 = 5$ to $t_2 = 15$ since this includes more time near the peak rate.

c) Words memorized $t_1 = 0$ to $t_2 = 10$: $\int_0^{10} -0.009t^2 + 0.18t \, dt = -0.003t^3 + 0.09t^2 \Big|_0^{10}$

$= -0.003(10^3) + 0.09(10^2) - 0$

$= -3 + 9 = \boxed{6}$

Words memorized $t_1 = 5$ to $t_2 = 15$: $\int_5^{15} -0.009t^2 + 0.18t \, dt = -0.003t^3 + 0.09t^2 \Big|_5^{15}$

$= -0.003(15^3) + 0.09(15^2) - [-0.003(5^3) + 0.09(5^2)]$

$= -10.125 + 20.25 - [-0.375 + 2.25]$

$= 10.125 - 1.875 = \boxed{8.25}$

(continued on the reverse)

Date: November 20, 2023.

2. For this problem, use the fact that $a(t) = v'(t) = s''(t)$. For both parts, assume constant acceleration.

a) A car accelerates from 0 to 100 kph in 10 seconds. How far did it travel over this time?

b) A car decelerates from 100 kph to a stop in 4 seconds. How far did it travel over this time?

a) $a(t) = \frac{100\text{ kph}}{10\text{ s}} = \frac{10}{60^2}\ \frac{\text{km}}{s^2}$ (using 60^2 sec $= 1$ hr)

$v(0) = 0$ (t measured in seconds)

$v(t) = \int a(t)\,dt = \frac{10}{60^2}t + C$ and $0 = v(0) = C$ so $v(t) = \frac{10}{60^2}t$

Distance traveled: $\int_0^{10} |v(t)|\,dt = \int_0^{10} \frac{10}{60^2}t\,dt = \frac{5}{60^2}t^2\Big|_0^{10} = \frac{500}{60^2}$ km

≈ 0.13889 km or 138.89 m

b) $a(t) = \frac{-100\text{ kph}}{4\text{ s}} = \frac{-25}{60^2}\ \frac{\text{km}}{s^2}$ and $v(0) = 100\ \frac{\text{km}}{\text{hr}} = \frac{100}{60^2}\ \frac{\text{km}}{s}$

$v(t) = \int a(t)\,dt = \frac{-25}{60^2}t + C$ and $\frac{100}{60^2} = v(0) = C.$

Hence $v(t) = -\frac{25}{60^2}t + \frac{100}{60^2}\ \frac{\text{km}}{s}$ (t measured in seconds)

Distance traveled: $\int_0^4 |v(t)|\,dt = \int_0^4 -\frac{25}{60^2}t + \frac{100}{60^2}\,dt$

$= -\frac{25}{2(60^2)}t^2 + \frac{100}{60^2}t\Big|_0^4$

$= \frac{-25(4^2)}{2(60^2)} + \frac{100(4)}{60^2} = \frac{200}{60^2} = \frac{1}{18}$ km

≈ 0.0556 km or 55.56 m

3. After WWII ended in 1945, the birth rate in the US increased dramatically (the baby boom). Suppose the birth rate $b(t)$ (in millions of births per year) for 1945 to 1965 was

$$b(t) = 4 + 0.025t$$

where t is the number of years after 1945.

a) According to this model, how many babies were born from 1945 to 1965?

b) The average value of $y = f(x)$ for $a \le x \le b$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$. Calculate the average number of births per year over the period 1945 to 1965.

c) Find the accumulation function $B(t)$ for the total number of births in the t years since 1945.

d) Starting after the war, how long did it take for 20 million births?

3. $b(t) = 4 + 0.025t$ ($t=0$ is 1945; applies until $t=20$ in 1965)

a) Total # born: $\int_0^{20} 4+0.025t\, dt = 4t + 0.0125t^2 \Big|_0^{20}$

$= 4(20) + 0.0125(20^2) - 0$

$= 85$ million

c) $B(t) = \int_0^t 4+0.025s\, ds = 4s + 0.0125s^2 \Big|_0^t = 4t + 0.0125t^2$

d) Solve $20 = B(t)$ for t.

$20 = 4t + 0.0125t^2 \Rightarrow t^2 + 320t - 1600 = 0$

$\Rightarrow t = \frac{-320 \pm \sqrt{320^2 + 4(1600)}}{2} = -160 \pm \frac{\sqrt{108800}}{2}$

Only positive t of interest: $t \approx 4.9242$ yrs

By 1950 (5 years) over 20 million had been born.

b) Average # births per year: $\frac{1}{20-0} \int_0^{20} 4+0.025t\, dt$

Same as part a

$= \frac{1}{20}(85)$

$= 4.25$ million/yr