SNAILS

1. Round Lake State Park is about an hour away in Idaho. A 2019 KREM2 news story ("North Idaho park uses free ice cream to battle invasive snails") reported on the seemingly sudden appearance of trapdoor snails in Round Lake.[1] These invasive snails are popular in aquariums, so it's likely that the original snails were just a few snails that were "set free." The snails had been spotted for a few years, but in 2019, ranger Mary McGraw said of the snail population: "It seems like it exploded." A small population that suddenly explodes is characteristic of exponential growth. Suppose the trapdoor snail population in Round Lake t years after their first introduction is modeled by the function $P(t) = 2e^{0.8t}$

a) How long did it take for the population to reach 1000?

b) The ice cream program reduced the snail population by about 500. If the program continues to remove 500 snails per year from Round Lake but the population in 2019 when the program started was 1000, then we can model the population t years after 2019 by the function $Q(t) = 1000e^{0.8t} - 500t$ What does this model predict will happen to the snail population? Use derivatives to figure this out.

c) How many snails would have to be removed each year to save round lake from trapdoor snails? That is, see if you can find a number C so that $1000e^{0.8t} - Ct \leq 0$ for some $t \geq 0$.

d) Is there a number C such that $\lim_{t\to\infty} 1000e^{0.8t} - Ct = 0$? Use Desmos to experiment.

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[1]Visitors who collected a dozen snails got a free ice cream sandwich. Twelve year-old Noah said of catching snails: "They get extremely big, and they go very slow... I like looking at snails a lot."

2. Exponential growth models (i.e. those satisfying the differential equation $P' = rP$ for some constant r) predict unbounded growth (provided $r > 0$). This isn't very realistic; more sophisticated models takes into account the carrying capacity of the environment. One such model is the **logistic** model:

$$P' = rP\left(1 - \frac{P}{L}\right)$$

where r is a growth rate constant and L is the (constant) carrying capacity. Using a modified version of an earlier Python project, we can explore this model for round lake. Start with an initial population of 2 snails, $r = 0.8$, and $L = 10,000$.

a) What is the population after 5 years? How does this compare with the prediction of the exponential model in problem 1a?

b) How long does it take the population to reach 1000? How does this compare with the prediction of the exponential model in problem 1a?

c) How does this model compare over the long-term with the exponential model?

d) What happens if the initial population is greater than L?

Challenge. Show that $P(t) = \dfrac{L}{1 + e^{-r(t-b)}}$ is a solution to the logistic model. In this case the constant b is given by $b = \frac{1}{r}\ln\left(\frac{L}{P(0)} - 1\right)$.