1. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to 1^+} \dfrac{x^2}{1-x^2}$

Solution. $-\infty$

2. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to\infty} \dfrac{6x^2-x+1}{2x^2+7x}$

Solution. 3

3. Is the function $f(\theta) = \begin{cases} \sin\theta & \text{if } \theta \le 0 \\ 1-\cos\theta & \text{if } \theta > 0 \end{cases}$ continuous at $x = 0$? Explain why or why not.

Solution. The function is continuous because $\lim_{\theta\to 0} \sin\theta = \lim_{\theta\to 0} 1 - \cos\theta$.

4. Use the definition of the derivative to find $f'(2)$ for $f(x) = (x-1)^2$.

Solution. $f'(2) = \lim_{h\to 0} \dfrac{(2+h-1)^2 - (2-1)^2}{h} = \lim_{h\to 0} \dfrac{h^2+2h+1-1}{h} = 2$

5. Is the function $f(x) = \dfrac{x+1}{x^2-1}$ continuous at $x = 1$? Explain why or why not.

Solution. The function is not defined at $x = 1$ and so cannot be continuous at $x = 1$.

6. Is the function $f(x) = |x-2|$ differentiable at $x = 2$? Explain why or why not.

Solution. The function is not differentiable at $x = 2$ because it has a cusp at $x = 2$.

7. Find the slope of the tangent line to the curve $y + y^3 = 2x^2 - 8$ at the point $(3, 2)$.

Solution. $\frac{12}{13}$

8. Find the second derivative of the function $f(x) = \cos(x^2)$.

Solution. $f''(x) = -2\sin(x^2) - 4x^2\cos(x^2)$

9. A particle moves along the curve $x^2 + y^2 = 25$. When the particle reaches the point $(3, 4)$ its x-coordinate is increasing at a rate of 8 m/s. At what rate is the y-coordinate changing at this moment?

Solution. -6 m/s

10. A cylinder with volume 64π cm^3, radius r, and height h is being crushed so that $\frac{dh}{dt} = -3$ cm/s (and its volume, given by $V = \pi r^2 h$, remains constant). Find $\frac{dr}{dt}$, the rate at which the radius is changing, when $r = 8$ cm.

Solution. 12 cm/s

11. Find the derivative of the function $g(x) = \displaystyle\int_1^{3x} t^2(1-t)^2\, dt$

Solution. $g'(x) = 3(3x)^2(1-3x)^2$

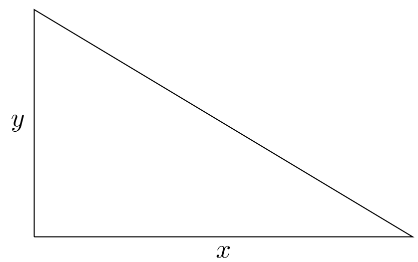
12. Use the Intermediate Value Theorem to show that the equation $2\left(x^3+17\right)^{\frac{1}{2}} - 9 = 0$ has a solution between -1 and 2.

Solution. Let $f(x) = 2\left(x^3+17\right)^{\frac{1}{2}} - 9$. The function f is continuous over $[-1, 2]$, $f(-1) < 0$, and $f(2) > 0$. Therefore by the IVT there is some c in $(-1, 2)$ such that $f(c) = 0$.

13. Find the absolute maximum and absolute minimum values of $f(x) = x - \sin(x)$ over the interval $[-\pi, \pi]$.

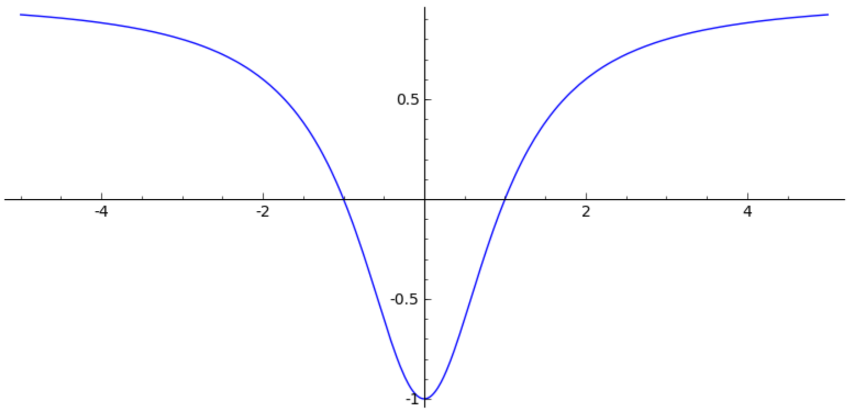
Solution. Max: (π, π). Min: $(-\pi, -\pi)$.

14. A right triangle has base length x and height y satisfying the equation $2x + y = 12$. Find the dimensions x and y that maximize the area of the triangle.



Solution. $x = 3$ and $y = 6$

15. Sketch the graph of $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$. Clearly indicate the location of all axis intercepts, asymptotes, and local extremes.



Solution.

16. The velocity of a particle at time t is given by the function $v(t) = 3 - 6t^2$ and after 1 second its position is $p(1) = 5$. Find an equation for the position of the particle at time t.

Solution. $p(t) = 3t - 2t^3 + 4$

17. Find the average value of the function $f(x) = (2 - x)^4$ over the interval $[0, 2]$.

Solution. $\frac{16}{5}$

18. Find the area above the curve $y = 2x + x^2$ **below** the x-axis.

Solution. $\frac{4}{3}$ ($-\frac{4}{3}$ is also acceptable).

19. The velocity of an object at time t is $v(t) = \dfrac{t}{2} - 1$. Find the total distance traveled from $t = 0$ to $t = 4$.

Solution. 2

20. Evaluate the integral $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\, dx$.

Solution. $2\sqrt{\sin x} + C$

21. Evaluate the integral $\displaystyle\int_2^3 (10 - 5x)^4\, dx$.

Solution. 125