1. Find the limit (either a number, $\infty$, or $-\infty$) or explain why it does not exist: \( \lim_{x \to 1^+} \frac{x^2}{1 - x^2} \)

Solution. \(-\infty\)

2. Find the limit (either a number, $\infty$, or $-\infty$) or explain why it does not exist: \( \lim_{x \to \infty} \frac{6x^2 - x + 1}{2x^2 + 7x} \)

Solution. 3

3. Is the function \( f(\theta) = \begin{cases} \sin \theta & \text{if } \theta \leq 0 \\ 1 - \cos \theta & \text{if } \theta > 0 \end{cases} \) continuous at \( x = 0 \)? Explain why or why not.

Solution. The function is continuous because \( \lim_{\theta \to 0} \sin \theta = \lim_{\theta \to 0} 1 - \cos \theta \).

4. Use the definition of the derivative to find \( f'(2) \) for \( f(x) = (x - 1)^2 \).

Solution. \( f'(2) = \lim_{h \to 0} \frac{(2 + h - 1)^2 - (2 - 1)^2}{h} = \lim_{h \to 0} \frac{h^2 + 2h + 1 - 1}{h} = 2 \)

5. Is the function \( f(x) = \frac{x + 1}{x^2 - 1} \) continuous at \( x = 1 \)? Explain why or why not.

Solution. The function is not defined at \( x = 1 \) and so cannot be continuous at \( x = 1 \).

6. Is the function \( f(x) = |x - 2| \) differentiable at \( x = 2 \)? Explain why or why not.

Solution. The function is not differentiable at \( x = 2 \) because it has a cusp at \( x = 2 \).

7. Find the slope of the tangent line to the curve \( y + y^3 = 2x^2 - 8 \) at the point \((3,2)\).

Solution. \( \frac{12}{13} \)

8. Find the second derivative of the function \( f(x) = \cos(x^2) \).

Solution. \( f''(x) = -2 \sin(x^2) - 4x^2 \cos(x^2) \)

9. A particle moves along the curve \( x^2 + y^2 = 25 \). When the particle reaches the point \((3,4)\) its \( x \)-coordinate is increasing at a rate of 8 m/s. At what rate is the \( y \)-coordinate changing at this moment?

Solution. -6 m/s

10. A cylinder with volume \( 64\pi \) cm\(^3\), radius \( r \), and height \( h \) is being crushed so that \( \frac{dh}{dt} = -3 \) cm/s (and its volume, given by \( V = \pi r^2 h \), remains constant). Find \( \frac{dr}{dt} \), the rate at which the radius is changing, when \( r = 8 \) cm.

Solution. 12 cm/s

11. Find the derivative of the function \( g(x) = \int_1^3 t^2(1-t)^2 \, dt \)

Solution. \( g'(x) = 3(3x)^2(1 - 3x)^2 \)

12. Use the Intermediate Value Theorem to show that the equation \( 2 \left( x^3 + 17 \right)^\frac{1}{2} - 9 = 0 \) has a solution between \(-1\) and \(2\).

Solution. Let \( f(x) = 2 \left( x^3 + 17 \right)^\frac{1}{2} - 9 \). The function \( f \) is continuous over \([-1,2]\), \( f(-1) < 0 \), and \( f(2) > 0 \). Therefore by the IVT there is some \( c \) in \((-1,2)\) such that \( f(c) = 0 \).

13. Find the absolute maximum and absolute minimum values of \( f(x) = x - \sin(x) \) over the interval \([-\pi, \pi]\).

Solution. Max: \((\pi, \pi)\). Min: \((-\pi, -\pi)\).
14. A right triangle has base length $x$ and height $y$ satisfying the equation $2x + y = 12$. Find the dimensions $x$ and $y$ that maximize the area of the triangle.

Solution. $x = 3$ and $y = 6$

15. Sketch the graph of $f(x) = \frac{x^2 - 1}{x^2 + 1}$. Clearly indicate the location of all axis intercepts, asymptotes, and local extremes.

Solution.

16. The velocity of a particle at time $t$ is given by the function $v(t) = 3 - 6t^2$ and after 1 second its position is $p(1) = 5$. Find an equation for the position of the particle at time $t$.

Solution. $p(t) = 3t - 2t^3 + 4$

17. Find the average value of the function $f(x) = (2 - x)^4$ over the interval $[0, 2]$.

Solution. $\frac{16}{5}$

18. Find the area above the curve $y = 2x + x^2$ below the $x$-axis.

Solution. $\frac{4}{3}$ (or $\frac{2}{3}$ is also acceptable).

19. The velocity of an object at time $t$ is $v(t) = \frac{t}{2} - 1$. Find the total distance traveled from $t = 0$ to $t = 4$.

Solution. 2

20. Evaluate the integral $\int \frac{\cos x}{\sqrt{\sin x}} \, dx$.

Solution. $2\sqrt{\sin x} + C$

21. Evaluate the integral $\int_{2}^{3} (10 - 5x)^{4} \, dx$.

Solution. 125