NAME(S):

A model rocket launched from the ground has an altitude (in meters) t seconds after launch of

$$a(t) = \begin{cases} 40t^2 & \text{if } t \leq 2 \\ 160 + 160(t-2) - 4(t-2)^2 & \text{if } t > 2 \end{cases}.$$

This is a piecewise function because the rocket engine stops 2 seconds into the flight, after which the rocket moves only under the influences of gravity and friction. We're going to start by considering just the first two seconds of flight.

1. a) The **average velocity** of the rocket over the first two seconds of its flight is the total change in altitude $(a(2)-a(0))$ divided by the total duration (2 seconds). Calculate this quantity.

b) The **average velocity** of the rocket between seconds 1 and 2 is $a(2)-a(1)$ divided by the duration (1 second). Calculate this quantity.

c) Calculate the **average velocity** of the rocket between $t = 1.5$ and $t = 2$ seconds.

2. Fill in the blanks in the following definitions.

Definition 1. The **average velocity** of the rocket between time t_0 and $t_0 + h$ is

$$\frac{a(\underline{\qquad}) - a(\underline{\qquad})}{h}$$

Definition 2. The **instantaneous velocity** of the rocket at time t_0 is the limit of the average velocity as h approaches 0:

$$\lim_{h \to 0} \left[\frac{a(\underline{\qquad}) - a(\underline{\qquad})}{h} \right].$$

3. Calculate the instantaneous velocity of the rocket at time $t_0 = 1$ using $a(t) = 40t^2$ and evaluating the limit. We'll call this number v_1. Do the same for time $t_0 = 2$ to get v_2.

4. Draw the curve $y = a(t)$ over the interval $[0, 2]$ carefully. Then plot the line passing through the point $(1, a(1))$ with slope v_1 and the line through the point $(2, a(2))$ with slope v_2. These are **tangent lines** to the curve at the points $(1, a(1))$ and $(2, a(2))$ (tangent comes from the Latin for "touching" and this should make sense after you've drawn the picture).

5. It is also possible to find a function for the instantaneous velocity of the rocket at time t. Use $a(t) = 40t^2$ in the formula for instantaneous velocity and evaluate the limit to find this function.