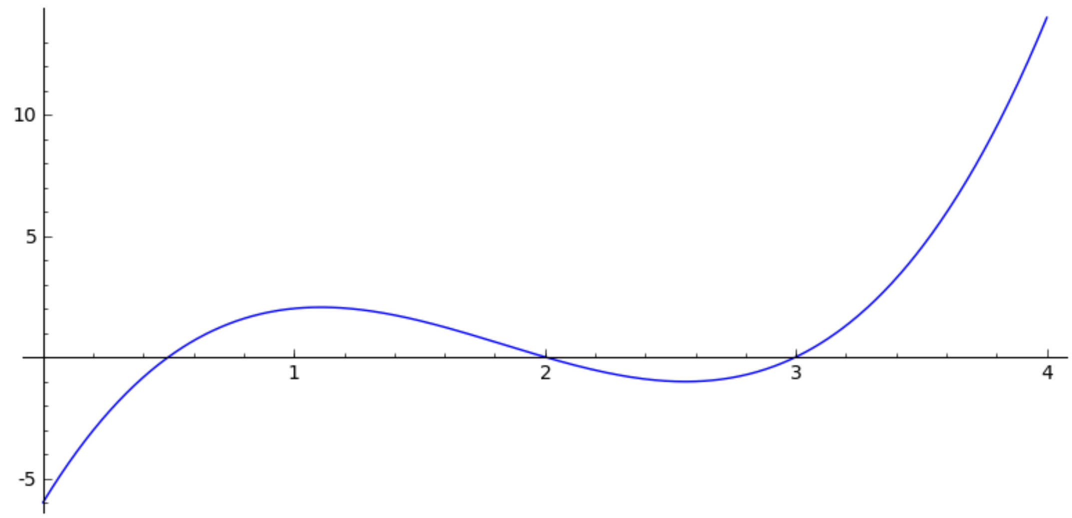

Figure 1: $y = 2t^3 - 11t^2 + 17t - 6$

1. Evaluate the indefinite integral $\int 2t^3 - 11t^2 + 17t - 6\ dt$.

Solution. $\frac{1}{2}t^4 - \frac{11}{3}t^3 + \frac{17}{2}t^2 - 6t + C$

2. Refer to the graph above while evaluating the following integrals.

a) $\int_0^{\frac{1}{2}} 2t^3 - 11t^2 + 17t - 6\ dt$

Solution. ≈ -1.302

b) $\int_0^{2} 2t^3 - 11t^2 + 17t - 6\ dt$

Solution. $\frac{2}{3}$

c) $\int_0^{3} 2t^3 - 11t^2 + 17t - 6\ dt$

Solution. 0

3. Define a new function $F(x) = \int_0^{x} 2t^3 - 11t^2 + 17t - 6\ dt$.

a) Try to identify the local extremes of $F(x)$ by interpreting $F(x)$ as a combination of areas under the graph in Figure 1. Skip this one and come back later it doesn't make sense now.

Solution. $F(x)$ must have a local minimum at $x = 0.5$ as the integral goes from adding area under the axis to adding area above the axis. For the same reason $F(x)$ has a local minimum at $x = 3$. $F(x)$ has a local maximum at $x = 2$ because the integral goes from adding area above the axis to area below the axis.

b) Evaluate the integral to find an expression for $F(x)$ that doesn't involve integration.

Solution. $F(x) = \frac{1}{2}x^4 - \frac{11}{3}x^3 + \frac{17}{2}x^2 - 6x$

c) Find the local extrema of $F(x)$ using the methods of chapter 3. Hint: Figure 1 may tell you how to factor $F'(x)$.

Solution. $F'(x) = 2x^3 - 11x^2 + 17x - 6 = (2x-1)(x-2)(x-3)$. A sign chart shows that the critical points at $x = \frac{1}{2}$ and $x = 3$ are local minima and the critical point at $x = 2$ is a local maximum.

4. Compare your answer for problem 1 with that for 3b. Compare $F'(x)$ with $2t^3 - 11t^2 + 17t - 6$.

5. The Fresnel function S is defined as $S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right)\, dt$. Do not try to evaluate this function as you did in 3b (because you can't).

a) Let $F(t)$ be an antiderivative of $f(t) = \sin\left(\frac{\pi t^2}{2}\right)$. Use the evaluation theorem to express $S(x)$ in terms of F.

Solution. $S(x) = F(x) - F(0)$

b) Differentiate your answer for part a to find $S'(x)$.

Solution. $S'(x) = F'(x) = f(x) = \sin\left(\frac{\pi x^2}{2}\right)$

6. Fill in the conclusion of the following theorem.

Theorem 1. *If f is continuous on $[a, b]$, then* $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$