1. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to -1} \dfrac{x+1}{x^2+1}$

Solution. 0

2. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to 0} \dfrac{|x|}{x}$

Solution. $\lim\limits_{x\to 0^-} \dfrac{|x|}{x} = -1 \neq 1 = \lim\limits_{x\to 0^+} \dfrac{|x|}{x}$ therefore $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.

3. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{h\to 0} \dfrac{(3+h)^2-9}{h}$

Solution. 6

4. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to \frac{\pi}{6}} \dfrac{1}{(\sin x)^2}$

Solution. 4

5. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to -\infty} \dfrac{3x+5}{x-4}$

Solution. 3

6. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to -\infty} 1-x^2$

Solution. $-\infty$

7. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to \infty} \sqrt{x^2+2x}-x$

Solution. 1

8. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to 5^-} \dfrac{x+1}{x-5}$

Solution. $-\infty$

9. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim\limits_{x\to 1^-} \dfrac{\sin\left(\frac{\pi}{2}x\right)}{1-x}$

Solution. ∞

10. Use the ϵ δ definition of the limit to prove that $\lim\limits_{x\to 1}(4x-5) = -1$.

Solution. Let $\epsilon > 0$. If $|x-1| < \frac{\epsilon}{4}$, then $4|x-1| < \epsilon$. It then follows that $|4x-4| < \epsilon$ and, consequently, $|4x-5-(-1)| < \epsilon$. Therefore by definition $\lim\limits_{x\to 1}(4x-5) = -1$.

11. Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and let $g(x) = x^2$. Find $\lim\limits_{x\to 0}(f\circ g)(x)$ or explain why the limit doesn't exist.

Solution. $(f\circ g)(x) = 1$ for all x. Therefore $\lim\limits_{x\to 0}(f\circ g)(x) = 1$.

12. Find all horizontal and vertical asymptotes of the function $f(x) = \dfrac{2x^2-2x}{x^2-1}$

Solution. HA: $y = 2$. VA: $x = -1$.

13. Find the value(s) of c that make the function continuous: $f(x) = \begin{cases} x^2 + c^2 & \text{if } x < 4 \\ 2cx & \text{if } x \geq 4 \end{cases}$

Solution. $c = 4$

14. Is the function $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$ continuous at $x = 1$? Explain why or why not.

Solution. $f(1) = 1 \neq 2 = \lim_{x \to 1} f(x)$ so the function is not continuous at $x = 1$.

15. Use the intermediate value theorem to show that the equation $x^4 - 4x^2 + 2 = 0$ has a solution.

Solution. Let $f(x) = x^4 - 4x^2 + 2$. The function f is continuous at all x and $f(0) = 2 > 0$ while $f(1) = -1 < 0$. Therefore by the IVT there is a number c in the interval $(0, 1)$ such that $f(c) = 0$. This c is a solution to the original equation.

16. Use the intermediate value theorem to show that the equation $\cos(x) - \sqrt{x} = 0$ has a solution in the interval $\left(0, \dfrac{\pi}{2}\right)$

Solution. Let $f(x) = \cos(x) - \sqrt{x}$. The function f is continuous on $[0, \infty)$ and $f(0) = 1 > 0$ while $f(\frac{\pi}{2}) = -\sqrt{\frac{\pi}{2}} < 0$. Therefore by the IVT there is a number c in the interval $(0, \frac{\pi}{2})$ such that $f(c) = 0$. This c is a solution to the original equation.