MATH 157 EXAM 1 PRACTICE SEPTEMBER 18, 2013

INSTRUCTIONS: Calculators, notes, cell phones, or other materials are not permitted. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown.

1. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to -1} \dfrac{x+1}{x^2+1}$

2. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to 0} \dfrac{|x|}{x}$

3. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{h\to 0} \dfrac{(3+h)^2-9}{h}$

4. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to \frac{\pi}{6}} \dfrac{1}{(\sin x)^2}$

5. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to -\infty} \dfrac{3x+5}{x-4}$

6. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to -\infty} 1-x^2$

7. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to \infty} \sqrt{x^2+2x}-x$

8. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to 5^-} \dfrac{x+1}{x-5}$

9. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to 1^-} \dfrac{\sin\left(\frac{\pi}{2}x\right)}{1-x}$

10. Use the ϵ δ definition of the limit to prove that $\lim_{x\to 1}(4x-5) = -1$.

11. Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and let $g(x) = x^2$. Find $\lim_{x\to 0}(f\circ g)(x)$ or explain why the limit doesn't exist.

12. Find all horizontal and vertical asymptotes of the function $f(x) = \dfrac{2x^2-2x}{x^2-1}$

13. Find the value(s) of c that make the function continuous: $f(x) = \begin{cases} x^2+c^2 & \text{if } x < 4 \\ 2cx & \text{if } x \geq 4 \end{cases}$

14. Is the function $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$ continuous at $x = 1$? Explain why or why not.

15. Use the intermediate value theorem to show that the equation $x^4 - 4x^2 + 2 = 0$ has a solution.

16. Use the intermediate value theorem to show that the equation $\cos(x) - \sqrt{x} = 0$ has a solution in the interval $\left(0, \dfrac{\pi}{2}\right)$