MATH 157 EXAM 1 SEPTEMBER 19, 2014

NAME:

INSTRUCTIONS: Calculators, notes, cell phones, and other other materials are not permitted. Additional scratch paper is available at the front of the class. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown. You may find the following useful:

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}$$

Definition. $\lim_{x\to a} f(x) = L$ if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that $|f(x)-L| < \epsilon$ whenever $0 < |x-a| < \delta$.

1. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to\infty}\dfrac{2x^2-x}{x^2-4}$

2. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to -1^+}\dfrac{x}{1-x^2}$

3. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to -2}\dfrac{x^2+3x+2}{x+2}$

4. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{h\to 0}\dfrac{\sqrt{4+h^2}-2}{h^2}$

5. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to 1}\dfrac{|1-x|}{x}$

6. Find the limit (either a number, ∞, or $-\infty$) or explain why it does not exist: $\lim_{x\to -\infty}\dfrac{x^2-x}{1-3x^3}$

7. Find all the vertical asymptotes of the function $f(x)=\dfrac{2x-4}{x^2-2x}$

8. If $\dfrac{1}{x^2+1} \le f(x) \le 1+x^2$ for all x in the interval $(-1,1)$, what is $\lim_{x\to 0} f(x)$?

9. On what interval(s) is the function $f(x)=\sqrt{1-x^2}$ continuous?

10. Is the function $f(x)=\begin{cases}\frac{\sin x}{2x} & \text{if } x\neq 0\\ 2 & \text{if } x=0\end{cases}$ continuous at $x=0$? Explain why or why not.

11. Find the value of c that makes the function f continuous: $f(x)=\begin{cases}2x-1 & \text{if } x\ge c\\ x^2 & \text{if } x<c\end{cases}$

12. Show that the equation $\sqrt{3-x}=x^3$ has a solution in the interval $[-1,2]$.

13. Use the ϵ–δ definition of the limit to prove that $\lim_{x\to 1}\dfrac{3x+5}{2}=4$.

14. Let $f(x)=\cos x$ and $g(x)=\begin{cases}\pi & \text{if } x<1\\ -\pi & \text{if } x\ge 1\end{cases}$. Is the composite function $f\circ g$ continuous at $x=1$? Explain why or why not.