

Definition 1. A function f is *continuous at a* if

$$\lim_{x \rightarrow a} f(x) = f(a).$$

It is important to realize that this means all 3 of the following things happen:

1. $f(a)$ is defined;
2. $\lim_{x \rightarrow a} f(x)$ exists;
3. $\lim_{x \rightarrow a} f(x) = f(a)$.

1. Explain why the following function is not continuous at $a = 2$.

a) $f(x) = \frac{1}{2 - 3x + x^2}$

b) $g(x) = \frac{2 - x}{2 - 3x + x^2}$

c) $h(x) = \begin{cases} \frac{2-x}{2-3x+x^2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

2. What value c would make the function continuous at $a = 0$?

$$\text{a) } f(x) = \begin{cases} c - x^2 & \text{if } x \geq 0 \\ \cos x & \text{if } x < 0 \end{cases}$$

$$\text{b) } g(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

3. Find the points at which the function $f(x) = \frac{1}{1 - \cos x}$ is not continuous.