

A model rocket launched from the ground has an altitude (in meters) t seconds after launch of

$$f(t) = \begin{cases} 40t^2 & \text{if } t \leq 2 \\ 160 + 160(t - 2) - 4(t - 2)^2 & \text{if } t > 2 \end{cases}.$$

This is a piecewise function because the rocket engine stops 2 seconds into the flight, after which the rocket moves only under the influences of gravity and friction. For now, consider just the first two seconds of flight (ignore the complicated part of the function).

Definition 1. The *average velocity* of the rocket between time t_0 and $t_0 + h$ is

$$\frac{f(t_0 + h) - f(t_0)}{h}$$

1. Calculate $f(0)$, $f(1)$, $f(1.5)$ (hint: $1.5 = \frac{3}{2}$), and $f(2)$ (you'll need these for the following problems).

2. a) Calculate the average velocity over the first 2 seconds of the rocket's flight.

b) Calculate the average velocity between $t = 1$ and $t = 2$ seconds.

c) Calculate the average velocity between $t = 1$ and $t = 1.5$ seconds.

Definition 2. The *instantaneous velocity* of the rocket at time t_0 is the limit of the average velocity as h approaches 0:

$$\lim_{h \rightarrow 0} \frac{f(t_0 + h) - f(t_0)}{h}$$

Note: this gives the *instantaneous rate of change* of whatever quantity is given by the function.

3. a) Calculate the instantaneous velocity of the rocket at time $t_0 = 1$ (call this number v_1).

b) Do the same for time $t_0 = 0$ to get v_0 .

4. Draw the curve $y = f(t)$ over the interval $[0, 2]$ carefully. Then plot the line passing through the point $(0, f(0))$ with slope v_0 and the line through the point $(1, f(1))$ with slope v_1 . These are **tangent lines** to the curve $y = f(t)$ at the points (tangent comes from the Latin for “touching” and this should make sense after you’ve drawn the picture).



5. It is also possible to find a function for the instantaneous velocity of the rocket at time t : use $f(t) = 40t^2$ in the formula for instantaneous velocity and evaluate the limit to find this function. (If you’ve already done this, then use your velocity function in the formula to find the instantaneous rate of change of velocity—also known as acceleration).