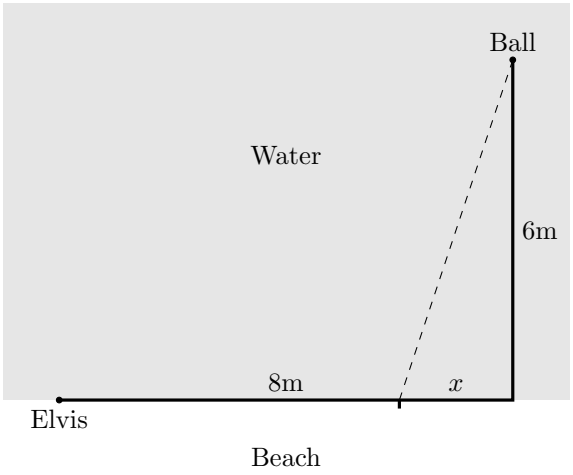
Name(s):

Math 157 Optimization October 31, 2014

1. A dog named Elvis is on the edge of a lake and his ball is in the water 8 meters down the shore and 6 meters into the water. The diagram shows the view from above. Elvis can run along the beach at a speed of 3 m/s and he can swim at 1 m/s. Elvis wants to get to the ball as quickly as possible.



a) How long does it take Elvis to get to the ball if he swims all the way?

b) How long does it take Elvis to get to the ball if he swims as little as possible?

c) Find an equation for the time it takes Elvis to get to the ball if he runs down the beach to a distance x from the point on the shore closest to the ball and then swims. Use this equation to find the shortest possible route to the ball.

2. Biologists have determined that if a fish swims at a speed v through the water, then its energy expenditure is proportional to v^3. Suppose that a hypothetical fish swims a distance of L meters against a fixed current of u meters per second. The energy expended by the fish is

$$E(v) = av^3 \left(\frac{L}{v-u} \right)$$

where a is a constant determined by the size and shape of the fish. What speed minimizes E? (Your answer will depend on u).