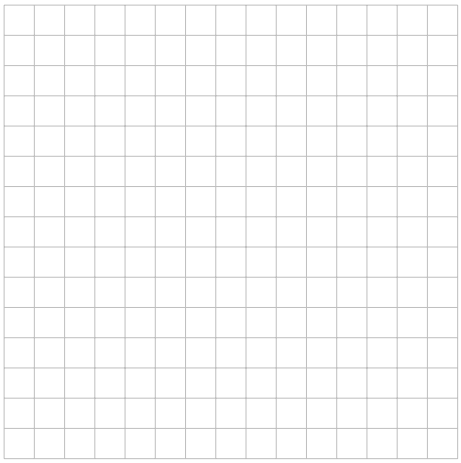
MATH 157 EXAM 3 APRIL 2, 2012

NAME:

INSTRUCTIONS: Calculators, notes, cell phones, or other materials are not permitted. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown.

1. Sketch the graph of a function that is continuous on the interval $[0, 5]$, has an absolute maximum at $x = 0$, and absolute minimum at $x = 4$, and critical points at $x = 1$ and $x = 3$.



2. Find the absolute maximum and absolute minimum values of $f(x) = (x^3 - 1)^2$ over the interval $[-2, 2]$.

3. Find the intervals of increase and the intervals of decrease of the function $f(x) = \dfrac{x^2}{x-4}$.

4. Find the intervals on which the function $f(x) = 1 - 3x - 24x^2 + x^4$ is concave up and those on which it is concave down.

5. Let $f(x) = \dfrac{x}{x^2 - 1}$. Use the Mean Value Theorem to show that $f'(x) = \frac{1}{3}$ for some x in the interval $[0, 2]$ or explain why the Mean Value Theorem does not apply.

6. Sketch the graph of $f(x) = 8x^2 - x^4$. Clearly indicate the location of all axis intercepts, asymptotes, and local extremes.

7. Sketch the graph of $f(x) = \dfrac{1}{x^2 - 2x}$. Clearly indicate the location of all axis intercepts, asymptotes, and local extremes.

8. Find the positive number x such that $f(x) = 4x^2 + \frac{1}{x}$ is a small as possible.

9. A cylindrical capsule with a total volume of 16π cm^3 has radius r and height h. What radius and height minimize the surface area of the capsule? Hint: The volume of the capsule is $\pi r^2 h$ and its surface area is $2\pi r^2 + 2\pi r h$.

10. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation of a solution to the equation $x^3 - 2 = 0$.

11. Find the general antiderivative of $f(x) = \dfrac{\sin x}{4} + x^{\frac{3}{4}}$

12. The velocity of a particle at time t is given by the function $v(t) = 3x^2 + 4x$ and after 1 second its position is $p(1) = 1$. Find an equation for the position of the particle at time t.