Ignoring the effects of friction, an object falling from a height of 64 feet under the influence of gravity will be $p(t) = 64 - 16t^2$ feet above the ground $t$ seconds after it begins falling. The first goal today is to figure out how fast the object is moving when it hits the ground.

1. Determine the time, $t_0$, at which the object hits the ground (i.e. reaches height 0).

2. Find the *average velocity* of the object by dividing the total change in height ($-64$, a negative number because the object is moving down) by the total duration of the fall (whatever value you found for $t_0$).

3. The average velocity of the object over the last $\Delta t$ seconds of its fall is

$$\frac{p(t_0 - \Delta t) - p(t_0)}{-\Delta t}.$$

Use this to find the average velocity over the last $\Delta t = 1$ and $\Delta t = \frac{1}{2}$ seconds (you should use the value you found for $t_0$ in #1).

4. The *instantaneous velocity* of the object when it hits the ground is exactly

$$\lim_{\Delta t \to 0} \left[ \frac{p(t_0 - \Delta t) - p(t_0)}{-\Delta t} \right].$$

Find this velocity.

5. The instantaneous velocity of the object after $t$ seconds is

$$v(t) = \lim_{\Delta t \to 0} \left[ \frac{p(t + \Delta t) - p(t)}{\Delta t} \right].$$

Find the function $v(t)$ and verify that $v(t_0)$ agrees with your answer to #4.

6. Draw the curve $y = p(t)$ carefully and then draw the line passing through the point $(1, p(1))$ with slope $v(1)$. This line is the *tangent line* for the curve at the point $(1, p(1))$ (tangent comes from the Latin for "touching" and this should make sense after you've drawn the picture).