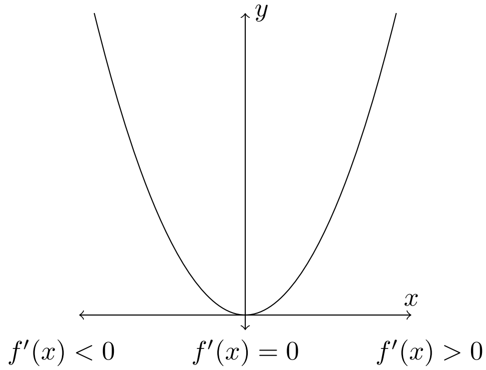
MATH 157 EXTREME VALUES 3/5/12

A function f has a *local maximum* at c if $f(c) \geq f(x)$ for every x close to c. The function has a *local minimum* at c if $f(c) \leq f(x)$ for every x close to c. A point $(x, f(x))$ that is either a local minimum or maximum is called a *local extreme* of the function.

For example, the function $f(x) = x^2$ has a local minimum at 0. The graph of this function is shown below along with some information about the derivative $f'(x) = 2x$.



1. Sketch the graph of $f(x) = \sin x$ from 0 to 2π below. Find any local extremes of f and indicate where $f'(x) < 0$, $f'(x) = 0$, and $f'(x) > 0$.

2. Sketch the graph of $f(x) = |x|$. Find any local extremes of f and indicate where $f'(x) < 0$, where $f'(x) > 0$, and where $f'(x)$ is undefined.

3. Make a conjecture by filling in the blanks: If f has a local extreme at a, then either $f'(a) =$_____ or $f'(a)$ ____________________. If $f(a)$ is a local maximum, then $f'(x)$ ______ for $x < a$ and $f'(x)$ ______ for $x > a$. If $f(a)$ is a local minimum, then $f'(x)$ ______ for $x < a$ and $f'(x)$ ______ for $x > a$.

After filling in the blanks, the first sentence of problem 3 should be: If f has a local extreme at a, then either $f'(a) = 0$ or $f'(a)$ does not exist. For this reason a number a for which $f'(a) = 0$ or $f'(a)$ does not exist is called a *critical number (or critical value)* for the function f.

4. Find all the critical numbers of $f(x) = 2x^3 + 3x^2 - 12x$. Determine which critical numbers are local maximums and which are local minimums.

5. Does the function $f(x) = 2x - 1$ have any critical numbers? Does it have any local extremes?

6. Are all critical numbers local extremes? Try to find a function that has critical numbers but no local extremes. Hint: if a is your critical number but f does not have an extreme at a, what must be true of $f'(x)$ for $x < a$ and for $x > a$?