Math 157 Exam 1

September 19, 2014

Name:

Instructions: Calculators, notes, cell phones, and other other materials are not permitted. Additional scratch paper is available at the front of the class. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown. You may find the following useful:

\[
\lim_{x \to 0} \frac{\sin x}{x} = 1
\]

Definition. \( \lim_{x \to a} f(x) = L \) if for every number \( \epsilon > 0 \) there is a number \( \delta > 0 \) such that \( |f(x) - L| < \epsilon \) whenever \( 0 < |x - a| < \delta \).

1. Find the limit (either a number, \( \infty \), or \( -\infty \)) or explain why it does not exist: \( \lim_{x \to \infty} \frac{2x^2 - x}{x^2 - 4} \)

2. Find the limit (either a number, \( \infty \), or \( -\infty \)) or explain why it does not exist: \( \lim_{x \to 1^+} \frac{x}{1 - x^2} \)

3. Find the limit (either a number, \( \infty \), or \( -\infty \)) or explain why it does not exist: \( \lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2} \)

4. Find the limit (either a number, \( \infty \), or \( -\infty \)) or explain why it does not exist: \( \lim_{h \to 0} \frac{\sqrt{4 + h^2} - 2}{h^2} \)

5. Find the limit (either a number, \( \infty \), or \( -\infty \)) or explain why it does not exist: \( \lim_{x \to 1} \frac{|1 - x|}{x} \)

6. Find the limit (either a number, \( \infty \), or \( -\infty \)) or explain why it does not exist: \( \lim_{x \to -\infty} \frac{x^2 - x}{1 - 3x^3} \)

7. Find all the vertical asymptotes of the function \( f(x) = \frac{2x - 4}{x^2 - 2x} \)

8. If \( \frac{1}{x^2 + 1} \leq f(x) \leq 1 + x^2 \) for all \( x \) in the interval \((-1, 1)\), what is \( \lim_{x \to 0} f(x) \)?

9. On what interval(s) is the function \( f(x) = \sqrt{1 - x^2} \) continuous?

10. Is the function \( f(x) = \begin{cases} \sin x & \text{if } x \neq 0 \\ \frac{x}{2} & \text{if } x = 0 \end{cases} \) continuous at \( x = 0 \)? Explain why or why not.

11. Find the value of \( c \) that makes the function \( f \) continuous: \( f(x) = \begin{cases} 2x - 1 & \text{if } x \geq c \\ x^2 & \text{if } x < c \end{cases} \)

12. Show that the equation \( \sqrt{3 - x} = x^3 \) has a solution in the interval \([-1, 2]\).

13. Use the \( \epsilon-\delta \) definition of the limit to prove that \( \lim_{x \to 1} \frac{3x + 5}{2} = 4 \).

14. Let \( f(x) = \cos x \) and \( g(x) = \begin{cases} \pi & \text{if } x < 1 \\ -\pi & \text{if } x \geq 1 \end{cases} \). Is the composite function \( f \circ g \) continuous at \( x = 1 \)? Explain why or why not.