CONTINUITY

Definition. A function f is continuous at c if $\lim_{x\to c} f(x) = f(c)$. This means that f is only continuous at c if all 3 of the following are true:

- (1) f(c) is defined,
- (2) $\lim_{x\to c} f(x)$ exists, and
- (3) $\lim_{x \to c} f(x) = f(c).$

1. Explain (with reference to the list above) why each of the following functions is **not** continuous at c = 2.

a)
$$f(x) = \frac{1}{2 - 3x + x^2}$$

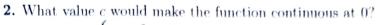
 $f(2)$ is not defined (item 1)
Also lin $f(x)$ DNE (item $x \neq 2$

b)
$$g(x) = \frac{2-x}{2-3x+x^2}$$

 $f(2)$ is not defined (iten 1)

Note
$$\frac{2-x}{2-3x+x^2} = \frac{2-x}{(2-x)(1-x)} = \frac{1}{1-x}$$
 if $x \neq 2$.
This means $\lim_{X \to 2} g(x) = \lim_{X \to 2} \frac{1}{1-x} = \frac{1}{1-x} = -1$
c) $h(x) = \begin{cases} \frac{2-x}{2-3x+x^2} & \text{if } x \neq 2\\ 1 & \text{if } x = 2 \end{cases}$
 $\lim_{X \to 2} h(x) = \lim_{X \to 2} \frac{2-x}{2-3x+x^2} = \lim_{X \to 2} \frac{1}{1-x} = -1$
 $\lim_{X \to 2} h(x) = \lim_{X \to 2} \frac{2-x}{2-3x+x^2} = \lim_{X \to 2} \frac{1}{1-x} = -1$
 $h(2) = 1 \neq -1$.
item 3 fails.

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a)
$$f(x) = \begin{cases} c - x^{2} & \text{if } x \ge 0 \\ \cos x & \text{if } x < 0 \end{cases}$$
Require $\lim_{x \to 0^{+}} (-x)^{2} = \lim_{x \to 0^{+}} (-x)^{2} =$

c) How does your answer to part b change if we restrict the domain of g to the interval $(0, \pi)$? Then the function is continuous everywhere (in the domain).

2,