CONTINUITY

Definition. A function f is **continuous** at c if $\boxed{\lim_{x\to c} f(x) = f(c)}$. This means that f is only continuous at c if all 3 of the following are true:

(1) $f(c)$ is defined,
(2) $\lim_{x\to c} f(x)$ exists, and
(3) $\lim_{x\to c} f(x) = f(c)$.

1. Explain (with reference to the list above) why each of the following functions is **not** continuous at $c = 2$.

a) $f(x) = \dfrac{1}{2 - 3x + x^2}$

b) $g(x) = \dfrac{2 - x}{2 - 3x + x^2}$

c) $h(x) = \begin{cases} \frac{2-x}{2-3x+x^2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

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2. What value c would make the function continuous at 0?

a) $f(x) = \begin{cases} c - x^2 & \text{if } x \geq 0 \\ \cos x & \text{if } x < 0 \end{cases}$

b) $g(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$

3. Find the points at which the function is **not** continuous.

a) $f(x) = \begin{cases} x & \text{if } x < -1 \\ x^3 & \text{if } -1 \leq x < 1 \\ \frac{1}{x} & \text{if } 1 \leq x \end{cases}$

b) $g(x) = \dfrac{1}{1 - \cos x}$

c) How does your answer to part b change if we restrict the domain of g to the interval $(0, \pi)$?