DERIVATIVES AND RATES OF CHANGE

The altitude of a model rocket (in meters) t seconds after launch is given by

$$f(t) = \begin{cases} 40t^2 & \text{if } t \leq 2 \\ 160 + 160(t-2) - 4(t-2)^2 & \text{if } t > 2 \end{cases}$$

This is a piecewise function because the rocket engine stops 2 seconds into the flight, after which the rocket moves only under the influences of gravity and friction.

1. Verify that the function $f(t)$ is continuous (otherwise it's a bad description of the rocket's flight).

Definition. The **average velocity** of the rocket between time t_1 and t_2 is

$$\frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

The **instantaneous velocity** of the rocket at time t_1 is the limit of the average velocity as t_2 approaches t_1:

$$v(t) = f'(t_1) = \lim_{t_2 \to t_1} \frac{f(t_2) - f(t_1)}{t_2 - t_1} = \lim_{h \to 0} \frac{f(t_1 + h) - f(t_1)}{h}$$

2. Use the definition of the derivative (above) to find $f'(t)$ for $t < 2$. (Yes, this is the hard way).

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3. Use differentiation rules to find $f'(t)$ for ...

a) ... $t < 2$ (check your answer for the previous problem)

b) ... $t > 2$

4. Does $f'(2)$ exist?

5. Differentiate your answers to problem 3 to find $f''(t)$, the acceleration of the rocket for ...

a) ... $t < 2$

b) ... $t > 2$

6. Does $f''(2)$ exist?

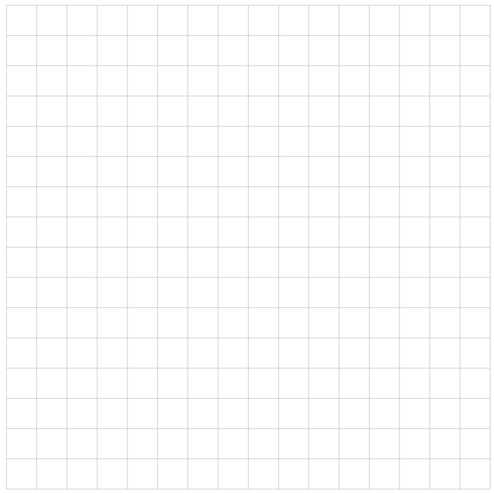
7. Differentiate your answers to problem 5 to find $f'''(t)$, the instantaneous rate of change of acceleration. Does your answer make sense? Explain why or why not.

8. Consider the function $g(x) = 2^x$.

a) Fill in the table and use the points to plot $y = g(x)$.

x	2^x
-2	
-1	
0	
1	
2	
3	

b) Use the graph $y = g(x)$ to make a rough estimate of $g'(x)$ and sketch $y = g'(x)$ it on the same axes.



Definition. The number e is the unique number such that $\frac{d}{dx}[e^x] = e^x$. The natural logarithm function $\ln x$ is the **inverse** of e^x:

$$\ln(e^x) = x \text{ and } e^{\ln x} = x$$

Theorem. *If c is constant, then* $\dfrac{d}{dx}[e^{cx}] = ce^{cx}$

9. Use the theorem and laws of exponents to solve the following.

a) Verify that $e^{x \ln 2} = 2^x$.

b) Use part a to calculate $\dfrac{d}{dx}[2^x]$.

c) Generalize to find a formula for $\dfrac{d}{dx}[a^x]$ (where a is a positive real constant).