CHAIN RULE AND IMPLICIT DIFFERENTIATION

1. Find the following derivatives. Recall that $\frac{d}{dx}[e^x] = e^x$ and $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$.

a) $\dfrac{d}{dx}\left[\tan^2 x\right]$

b) $\dfrac{d}{dx}\left[\tan^2(x^2)\right]$

c) $\dfrac{d}{dx}\left[e^{\tan^2 x}\right]$

d) $\dfrac{d}{dx}\left[e^{\tan^2(x^2)}\right]$

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Sometimes a function $y = f(x)$ is defined implicitly by an equation involving x and y. For example, the equation for the unit circle $x^2 + y^2 = 1$ implicitly defines the functions $y = \sqrt{1-x^2}$ and $y = -\sqrt{1-x^2}$. It's often more convenient to leave the functions implicit (and sometimes it's not possible to solve for y). The chain rule allows us to find the derivative $\frac{dy}{dx}$ of such implicit functions: differentiate both sides of the equation and apply the chain rule as needed (remembering that y is a function of x). In the example of the circle $x^2 + y^2 = 1$:

$$\begin{aligned} \frac{d}{dx}\left[x^2 + y^2\right] &= \frac{d}{dx}[1] \\ \frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[y^2\right] &= 0 \\ 2x + 2y\frac{dy}{dx} &= 0 \text{ (using the chain rule)} \end{aligned}$$

We may then solve for $\boxed{\frac{dy}{dx} = -\frac{x}{y}}$.

2. Use implicit differentiation to find $\frac{dy}{dx}$:

a) $e^y = x$

b) $e^{(y^2)} = x^2$

c) $\sin(y) = x$

d) $x^2 - xy + y^2 = 1$