INVERSE TRIG FUNCTIONS

1. Evaluate the following:

a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

c) $\tan^{-1}(1) = \frac{\pi}{4}$

d) $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}$

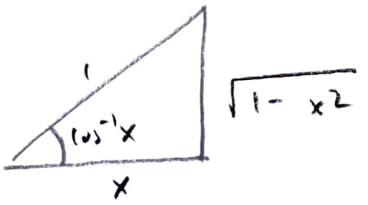
e) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

f) $\tan^{-1}(\tan(\pi)) = 0$

Date: October 11, 2021.

2. Use implicit differentiation and the triangle trick from last class to find $\frac{d}{dx}[\cos^{-1} x]$.

$y = \cos^{-1} x \Rightarrow \cos y = x \Rightarrow y'(-\sin y) = 1 \Rightarrow y' = -\frac{1}{\sin y} = -\frac{1}{\sin(\cos^{-1} x)} = -\frac{1}{\sqrt{1-x^2}}$



3. Find the following derivatives.

a) $\frac{d}{dx}\left[\cos^{-1}\left(\frac{1}{x}\right)\right] = \frac{-1}{\sqrt{1-(\frac{1}{x})^2}}\left(-\frac{1}{x^2}\right) = \frac{1}{x^2\sqrt{1-(\frac{1}{x})^2}} = \frac{1}{x\sqrt{x^2-1}}$

Connection with $\frac{d}{dx}[\sec^{-1} x]$?

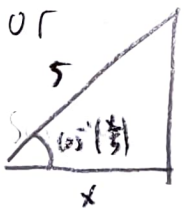
$\cos y = \frac{1}{x} \Rightarrow \sec y = x$

b) $\frac{d}{dx}[\tan^{-1}(x^2)] = \frac{1}{1+(x^2)^2}(2x) = \frac{2x}{1+x^4}$

c) $\frac{d}{dx}[x\sin^{-1}(5x)] = \sin^{-1}(5x) + x\left[\frac{5}{\sqrt{1-(5x)^2}}\right]$

d) $\frac{d}{dx}\left[\tan\left(\cos^{-1}\left(\frac{x}{5}\right)\right)\right] = \sec^2\left(\cos^{-1}\left(\frac{x}{5}\right)\right)\left[\frac{-1/5}{\sqrt{1-(\frac{x}{5})^2}}\right] \ldots$

OR



$\tan\left(\cos^{-1}\left(\frac{x}{5}\right)\right) = \frac{\sqrt{25-x^2}}{x} = \sqrt{\frac{25}{x^2}-1}$

$\frac{d}{dx}\left[\tan\left(\cos^{-1}\left(\frac{x}{5}\right)\right)\right] = \frac{d}{dx}\left[\sqrt{\frac{25}{x^2}-1}\right] = \frac{1}{2}\left(\frac{25}{x^2}-1\right)^{-\frac{1}{2}}\left(\frac{-50}{x^3}\right)$

Challenge. Use implicit differentiation and the triangle trick from last class to find $\frac{d}{dx}[\sec^{-1} x]$.