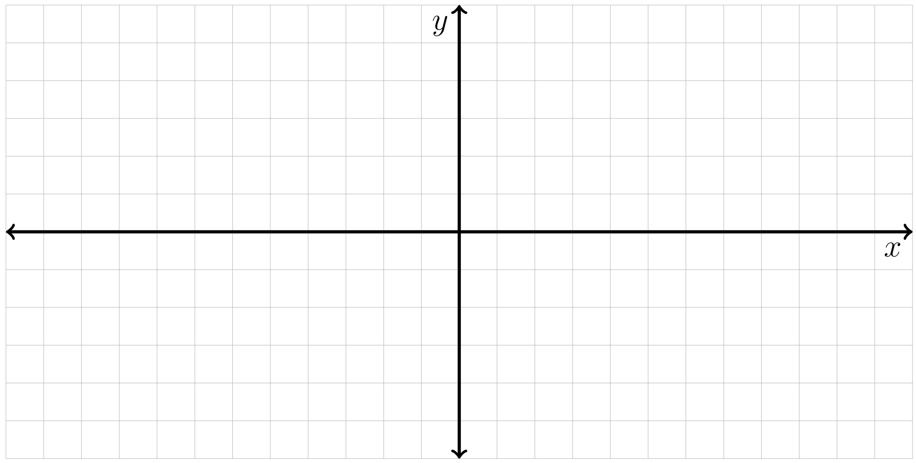
EXTREMES

Definition. The number a (in the interior of the domain of f) is a **critical number** for $f(x)$ if $f'(a) = 0$ or $f'(a)$ does not exist.

1. Sketch the graph of a function that has a local maximum at $x = -5$, a critical number that is not a local extreme at $x = -1$, and local minimum at $x = 3$.



2. Find the critical numbers of the function:

$$g(x) = 3x^4 - 8x^3 + 6x^2 - 2$$

3. Find the critical numbers of the function (it may help to sketch a graph of the function):

$$f(x) = |x^2 - 2x|$$

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Theorem (Extreme Value Theorem). *If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.*

Method. To find the absolute extreme values of a continuous function f over the interval $[a, b]$:

1. Find the critical numbers for f **that lie in the interval** $[a, b]$.
2. Evaluate the function at these critical numbers and at a and b.
3. The largest value obtained in the previous step is the absolute maximum, the smallest is the absolute minimum.

4. Find the absolute maximum and minimum values of the function $g(x) = 3x^4 - 8x^3 + 6x^2 - 2$ over the interval $[-1, 2]$.

5. Find the absolute maximum and minimum values of the function $f(x) = |x^2 - 2x|$ over the interval $[0.5, 4]$.

Challenge. Prove that $f(x) = x^3 + x^2 + x + 1$ has no local extremes.