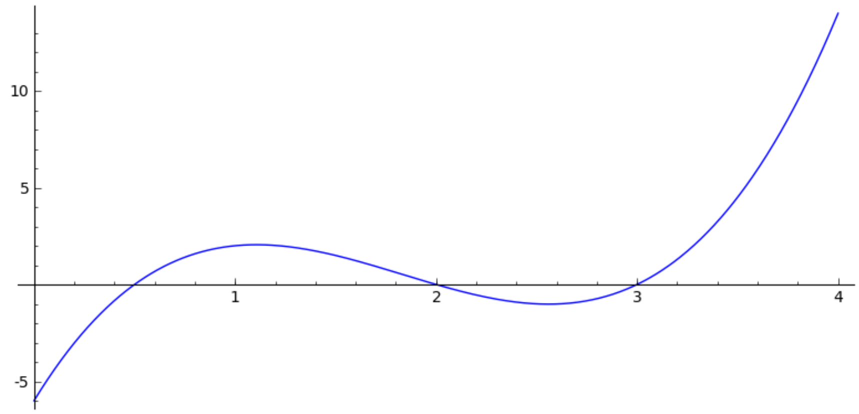
INTEGRATION (AND THE FUNDAMENTAL THEOREM OF CALCULUS)

Theorem (FTC I). *If f is continuous, then* $\frac{d}{dx}\int_a^x f(t)\ dt = f(x)$.



$y = 2t^3 - 11t^2 + 17t - 6$

1. Use the graph above to place the definite integrals in order (by their values) without evaluating the integrals:

a) $\int_0^{\frac{1}{2}} 2t^3 - 11t^2 + 17t - 6\ dt$

b) $\int_0^{1} 2t^3 - 11t^2 + 17t - 6\ dt$

c) $\int_0^{2} 2t^3 - 11t^2 + 17t - 6\ dt$

d) $\int_0^{4} 2t^3 - 11t^2 + 17t - 6\ dt$

2. Define a new function $\boxed{F(x) = \int_0^x 2t^3 - 11t^2 + 17t - 6\ dt}$

a) Try to identify the local extremes of $F(x)$ by interpreting $F(x)$ as a combination of areas under the graph in Figure 1. Recall that local extremes occur when F switches from increasing to decreasing or from decreasing to increasing.

b) Find the local extremes of $F(x)$ using the methods of chapter 3. Hint: the graph $y = 2t^3 - 11t^2 + 17t - 6$ crosses the x-axis at $t = \frac{1}{2}$, $t = 2$, and $t = 3$, hence $2t^3 - 11t^2 + 17t - 6$ has factors $2t - 1$, $t - 2$ and $t - 3$. Use the graph to determine signs.

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3. The Fresnel function S is defined as $S(x) = \int_0^x \sin\left(t^2\right)\, dt$. Do not try to evaluate this integral to find an expression for $S(x)$ (because it's impossible). Fresnel functions first arose in optics, specifically diffraction. Find the location of a local maximum of S and a local minimum of S.

4. Evaluate the following indefinite integrals. (Check answers by differentiating.)

a) $\int 2x \sin(x^2)\; dx$

b) $\int (6x^3 + 3x)\sqrt{x^4 + x^2 + 1}\; dx$

c) $\int (\sin x) e^{\cos x} dx$