

Basic Algebra Formulas

Arithmetic Operations

$$a(b + c) = ab + ac, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \quad \frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}$$

Laws of Signs

$$-(-a) = a, \quad \frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

Zero Division by zero is not defined.

$$\text{If } a \neq 0: \quad \frac{0}{a} = 0, \quad a^0 = 1, \quad 0^a = 0$$

$$\text{For any number } a: a \cdot 0 = 0 \cdot a = 0$$

Laws of Exponents

$$a^m a^n = a^{m+n}, \quad (ab)^m = a^m b^m, \quad (a^m)^n = a^{mn}, \quad a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

If $a \neq 0$, then

$$\frac{a^m}{a^n} = a^{m-n}, \quad a^0 = 1, \quad a^{-m} = \frac{1}{a^m}.$$

The Binomial Theorem For any positive integer n ,

$$\begin{aligned} (a + b)^n &= a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2} a^{n-2}b^2 \\ &\quad + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a^{n-3}b^3 + \cdots + nab^{n-1} + b^n. \end{aligned}$$

For instance,

$$\begin{aligned} (a + b)^2 &= a^2 + 2ab + b^2, & (a - b)^2 &= a^2 - 2ab + b^2 \\ (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3, & (a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3. \end{aligned}$$

Factoring the Difference of Like Integer Powers, $n > 1$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$$

For instance,

$$\begin{aligned} a^2 - b^2 &= (a - b)(a + b), \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2), \\ a^4 - b^4 &= (a - b)(a^3 + a^2b + ab^2 + b^3). \end{aligned}$$

Completing the Square If $a \neq 0$, then

$$ax^2 + bx + c = au^2 + C \quad \left(u = x + (b/2a), C = c - \frac{b^2}{4a} \right)$$

The Quadratic Formula

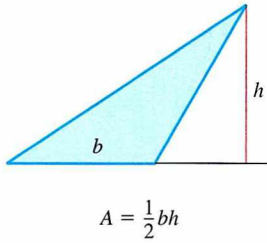
If $a \neq 0$ and $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

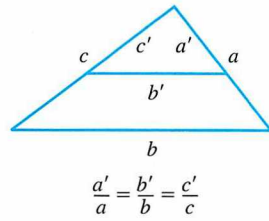
Geometry Formulas

A = area, B = area of base, C = circumference, S = surface area, V = volume

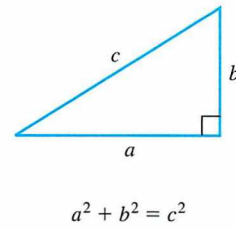
Triangle



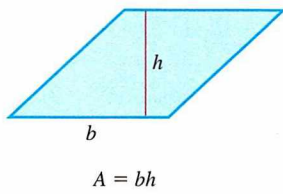
Similar Triangles



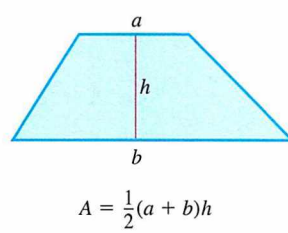
Pythagorean Theorem



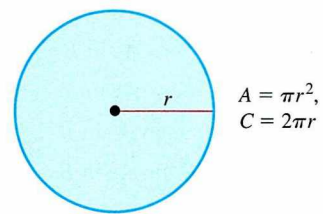
Parallelogram



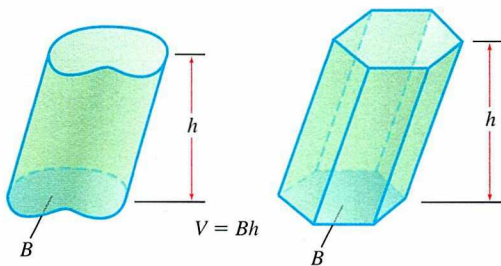
Trapezoid



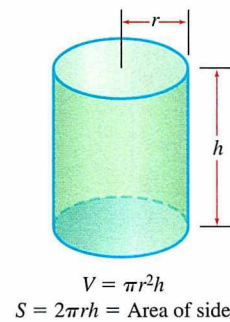
Circle



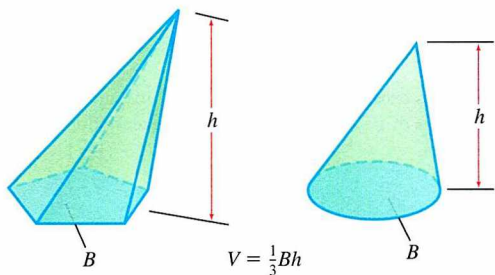
Any Cylinder or Prism with Parallel Bases



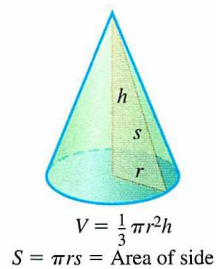
Right Circular Cylinder



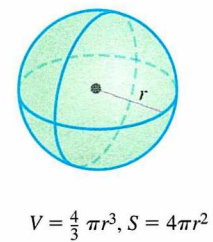
Any Cone or Pyramid



Right Circular Cone



Sphere



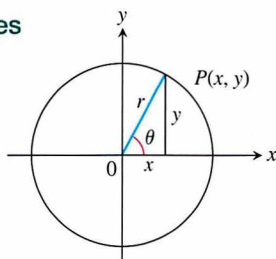
Trigonometry Formulas

Definitions and Fundamental Identities

Sine: $\sin \theta = \frac{y}{r} = \frac{1}{\csc \theta}$

Cosine: $\cos \theta = \frac{x}{r} = \frac{1}{\sec \theta}$

Tangent: $\tan \theta = \frac{y}{x} = \frac{1}{\cot \theta}$



Identities

$\sin(-\theta) = -\sin \theta$, $\cos(-\theta) = \cos \theta$

$\sin^2 \theta + \cos^2 \theta = 1$, $\sec^2 \theta = 1 + \tan^2 \theta$, $\csc^2 \theta = 1 + \cot^2 \theta$

$\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin\left(A - \frac{\pi}{2}\right) = -\cos A, \quad \cos\left(A - \frac{\pi}{2}\right) = \sin A$$

$$\sin\left(A + \frac{\pi}{2}\right) = \cos A, \quad \cos\left(A + \frac{\pi}{2}\right) = -\sin A$$

$$\sin A \sin B = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$$

$$\cos A \cos B = \frac{1}{2} \cos(A - B) + \frac{1}{2} \cos(A + B)$$

$$\sin A \cos B = \frac{1}{2} \sin(A - B) + \frac{1}{2} \sin(A + B)$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

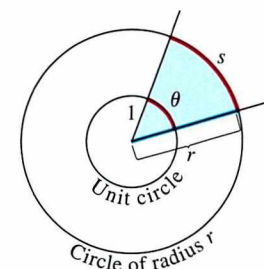
$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

Trigonometric Functions

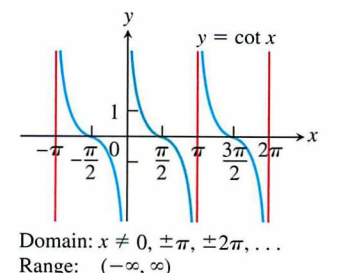
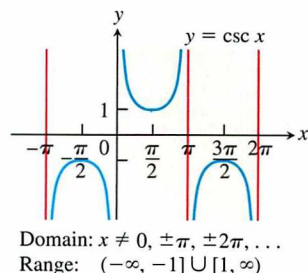
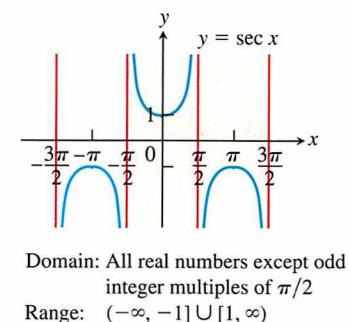
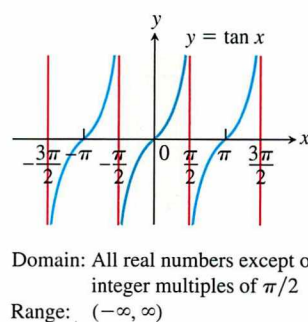
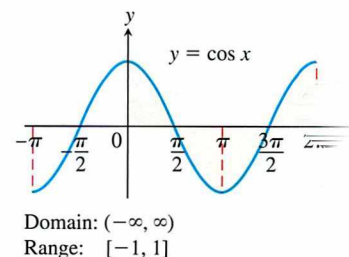
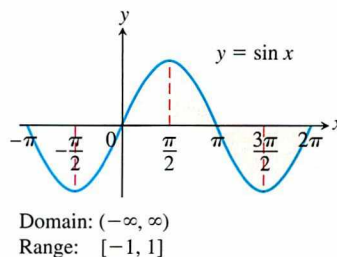
Radian Measure



$\frac{s}{r} = \frac{\theta}{1} = \theta$ or $\theta = \frac{s}{r}$,
 $180^\circ = \pi$ radians.

Degrees	Radians

The angles of two common triangles, in degrees and radians.



Series

Tests for Convergence of Infinite Series

- 1. The n th-Term Test:** Unless $a_n \rightarrow 0$, the series diverges.
- 2. Geometric series:** $\sum ar^n$ converges if $|r| < 1$; otherwise it diverges.
- 3. p -series:** $\sum 1/n^p$ converges if $p > 1$; otherwise it diverges.
- 4. Series with nonnegative terms:** Try the Integral Test, Ratio Test, or Root Test. Try comparing to a known series with the Comparison Test or the Limit Comparison Test.
- 5. Series with some negative terms:** Does $\sum |a_n|$ converge? If yes, so does $\sum a_n$ because absolute convergence implies convergence.
- 6. Alternating series:** $\sum a_n$ converges if the series satisfies the conditions of the Alternating Series Test.

Taylor Series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \quad -1 < x \leq 1$$

$$\ln \frac{1+x}{1-x} = 2 \tanh^{-1} x = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + \cdots \right) = 2 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, \quad |x| < 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \leq 1$$

Binomial Series

$$\begin{aligned} (1+x)^m &= 1 + mx + \frac{m(m-1)x^2}{2!} + \frac{m(m-1)(m-2)x^3}{3!} + \cdots + \frac{m(m-1)(m-2)\cdots(m-k+1)x^k}{k!} + \cdots \\ &= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad |x| < 1, \end{aligned}$$

where

$$\binom{m}{1} = m, \quad \binom{m}{2} = \frac{m(m-1)}{2!}, \quad \binom{m}{k} = \frac{m(m-1)\cdots(m-k+1)}{k!} \quad \text{for } k \geq 3.$$

Vector Operator Formulas (Cartesian Form)

Formulas for Grad, Div, Curl, and the Laplacian

Cartesian (x, y, z) $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the directions of increasing x , y , and z . M , N , and P are the scalar components of $\mathbf{F}(x, y, z)$ in these directions.

Gradient $\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$

Divergence $\nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$

Laplacian $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Vector Triple Products

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = (\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u} = (\mathbf{w} \times \mathbf{u}) \cdot \mathbf{v}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$$

The Fundamental Theorem of Line Integrals

Part 1 Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ be a vector field whose components are continuous throughout an open connected region D in space. Then there exists a differentiable function f such that

$$\mathbf{F} = \nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

if and only if for all points A and B in D , the value of $\int_A^B \mathbf{F} \cdot d\mathbf{r}$ is independent of the path joining A to B in D .

Part 2 If the integral is independent of the path from A to B , its value is

$$\int_A^B \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A).$$

Green's Theorem and Its Generalization to Three Dimensions

Tangential form of Green's Theorem: $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} \, dA$

Stokes' Theorem: $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma$

Normal form of Green's Theorem: $\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R (\nabla \cdot \mathbf{F}) \, dA$

Divergence Theorem: $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_D \nabla \cdot \mathbf{F} \, dV$

Vector Identities

In the identities here, f and g are differentiable scalar functions; $\mathbf{F}$, $\mathbf{F}_1$, and $\mathbf{F}_2$ are differentiable vector fields; and a and b are real constants.

$$\nabla \times (\nabla f) = \mathbf{0}$$

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla \cdot (g\mathbf{F}) = g\nabla \cdot \mathbf{F} + \nabla g \cdot \mathbf{F}$$

$$\nabla \times (g\mathbf{F}) = g\nabla \times \mathbf{F} + \nabla g \times \mathbf{F}$$

$$\nabla \cdot (a\mathbf{F}_1 + b\mathbf{F}_2) = a\nabla \cdot \mathbf{F}_1 + b\nabla \cdot \mathbf{F}_2$$

$$\nabla \times (a\mathbf{F}_1 + b\mathbf{F}_2) = a\nabla \times \mathbf{F}_1 + b\nabla \times \mathbf{F}_2$$

$$\nabla(\mathbf{F}_1 \cdot \mathbf{F}_2) = (\mathbf{F}_1 \cdot \nabla)\mathbf{F}_2 + (\mathbf{F}_2 \cdot \nabla)\mathbf{F}_1 +$$

$$\mathbf{F}_1 \times (\nabla \times \mathbf{F}_2) + \mathbf{F}_2 \times (\nabla \times \mathbf{F}_1)$$

$$\nabla \cdot (\mathbf{F}_1 \times \mathbf{F}_2) = \mathbf{F}_2 \cdot (\nabla \times \mathbf{F}_1) - \mathbf{F}_1 \cdot (\nabla \times \mathbf{F}_2)$$

$$\nabla \times (\mathbf{F}_1 \times \mathbf{F}_2) = (\mathbf{F}_2 \cdot \nabla)\mathbf{F}_1 - (\mathbf{F}_1 \cdot \nabla)\mathbf{F}_2 +$$

$$(\nabla \cdot \mathbf{F}_2)\mathbf{F}_1 - (\nabla \cdot \mathbf{F}_1)\mathbf{F}_2$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - (\nabla \cdot \nabla)\mathbf{F} = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$(\nabla \times \mathbf{F}) \times \mathbf{F} = (\mathbf{F} \cdot \nabla)\mathbf{F} - \frac{1}{2}\nabla(\mathbf{F} \cdot \mathbf{F})$$

Limits

General Laws

If L , M , c , and k are real numbers and

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = M, \quad \text{then}$$

Sum Rule: $\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$

Difference Rule: $\lim_{x \rightarrow c} (f(x) - g(x)) = L - M$

Product Rule: $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = L \cdot M$

Constant Multiple Rule: $\lim_{x \rightarrow c} (k \cdot f(x)) = k \cdot L$

Quotient Rule: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$

The Sandwich Theorem

If $g(x) \leq f(x) \leq h(x)$ in an open interval containing c , except possibly at $x = c$, and if

$$\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L,$$

then $\lim_{x \rightarrow c} f(x) = L$.

Inequalities

If $f(x) \leq g(x)$ in an open interval containing c , except possibly at $x = c$, and both limits exist, then

$$\lim_{x \rightarrow c} f(x) \leq \lim_{x \rightarrow c} g(x).$$

Continuity

If g is continuous at L and $\lim_{x \rightarrow c} f(x) = L$, then

$$\lim_{x \rightarrow c} g(f(x)) = g(L).$$

Specific Formulas

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$, then

$$\lim_{x \rightarrow c} P(x) = P(c) = a_n c^n + a_{n-1} c^{n-1} + \cdots + a_0.$$

If $P(x)$ and $Q(x)$ are polynomials and $Q(c) \neq 0$, then

$$\lim_{x \rightarrow c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

If $f(x)$ is continuous at $x = c$, then

$$\lim_{x \rightarrow c} f(x) = f(c).$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

L'Hôpital's Rule

If $f(a) = g(a) = 0$, both f' and g' exist in an open interval I containing a , and $g'(x) \neq 0$ on I if $x \neq a$, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)},$$

assuming the limit on the right side exists.

Differentiation Rules

General Formulas

Assume u and v are differentiable functions of x .

Constant: $\frac{d}{dx}(c) = 0$

Sum: $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Difference: $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

Constant Multiple: $\frac{d}{dx}(cu) = c \frac{du}{dx}$

Product: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$

Quotient: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Power: $\frac{d}{dx}x^n = nx^{n-1}$

Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Exponential and Logarithmic Functions

$$\frac{d}{dx}e^x = e^x \quad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}a^x = a^x \ln a \quad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2} \quad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x \quad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \quad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x \quad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \quad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \quad \frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2} \quad \frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1+x^2}}$$

Parametric Equations

If $x = f(t)$ and $y = g(t)$ are differentiable, then

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$

Integration Rules

General Formulas

Zero: $\int_a^a f(x) dx = 0$

Order of Integration: $\int_b^a f(x) dx = -\int_a^b f(x) dx$

Constant Multiples: $\int_a^b kf(x) dx = k \int_a^b f(x) dx, \quad k \text{ any number}$

$$\int_a^b -f(x) dx = -\int_a^b f(x) dx, \quad k = -1$$

Sums and Differences: $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

Additivity: $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

Max-Min Inequality: If $\max f$ and $\min f$ are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \leq \int_a^b f(x) dx \leq \max f \cdot (b - a).$$

Domination: $f(x) \geq g(x) \quad \text{on} \quad [a, b] \quad \text{implies} \quad \int_a^b f(x) dx \geq \int_a^b g(x) dx$

$$f(x) \geq 0 \quad \text{on} \quad [a, b] \quad \text{implies} \quad \int_a^b f(x) dx \geq 0$$

The Fundamental Theorem of Calculus

Part 1 If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t) dt$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is $f(x)$:

$$F'(x) = \frac{d}{dx} \int_a^x f(t) dt = f(x).$$

Part 2 If f is continuous at every point of $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a).$$

Substitution in Definite Integrals

$$\int_a^b f(g(x)) \cdot g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Integration by Parts

$$\int_a^b u(x) v'(x) dx = u(x) v(x) \Big|_a^b - \int_a^b v(x) u'(x) dx$$

A Brief Table of Integrals follows the Index at the back of the text.