Math 258 Exam 4 December 3, 2012

Name:

Instructions: Answer the questions in the space provided. Show all your work: even a correct answer may receive little or no credit if a method of solution is not shown. Always specify which tests for convergence or divergence you are using and, where appropriate, which series are being used for comparisons.

Some important series and their radii of convergence:

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \dots$	$R = 1$

1. Determine if the sequence $\left\{\cos\left(\frac{\pi}{n}\right)\right\}_{n=1}^{\infty}$ converges or diverges. If it converges, find its limit.

2. Determine whether the series $\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n}$ is convergent or divergent. If it is convergent, find its sum.

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+2}$ is convergent or divergent.

4. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$ is convergent or divergent.

5. Determine whether the series $\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{\sqrt{n+1}}$ is absolutely convergent, conditionally convergent, or divergent.

6. Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n x^{2n}}{n!}$.

7. Find a power series representation of the function $f(x) = \frac{1}{8 - x^3}$ and determine the interval of convergence.

8. Use the binomial series to evaluate $\int \sqrt{1+x^2}\ dx$ as a series. Either write your answer using a $\sum$ or find the first 4 terms of the series. Do not simplify your answer.

9. Find the Taylor series for $f(x) = \ln x$ at $a = 1$. Either write your answer using a $\sum$ or find the first 5 terms of the series (i. e. find c_0, c_1, c_2, c_3, and c_4 where $\ln x = c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + c_4(x-1)^4 + \dots$).