MATH 258 EXAM 3 NOVEMBER 1, 2012

NAME: Solutions

INSTRUCTIONS: Answer the questions in the space provided. Show all your work: even a correct answer may receive little or no credit if a method of solution is not shown. Water has a density of $1000\frac{\text{kg}}{\text{m}^3}$. The acceleration of gravity is $9.8\frac{\text{m}}{\text{s}^2}$.

1. Find the area of the region enclosed by the curve $y = x^2 - 2x - 1$ and the line $y = x - 1$.

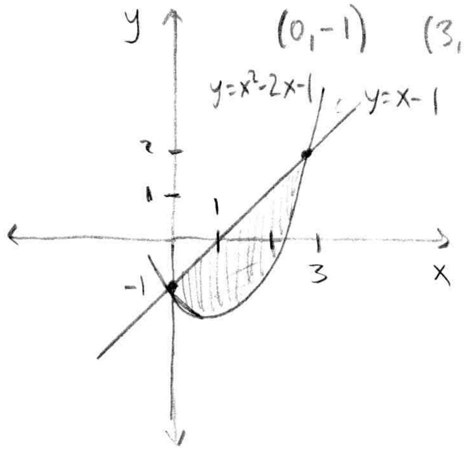
Intersections: $x^2-2x-1 = x-1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ or $x = 3$

$(0,-1)$ $(3,2)$



Area: $\int_0^3 x-1-(x^2-2x-1)\,dx$

$= \int_0^3 -x^2+3x\,dx$

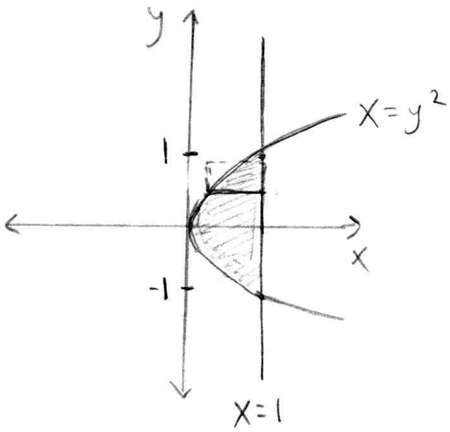
$= -\frac{1}{3}x^3 + \frac{3}{2}x^2 \Big|_0^3$

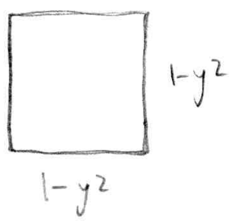
$= -9 + \frac{27}{2}$

$= \frac{-18+27}{2}$

$= \boxed{\frac{9}{2}}$

2. The base of a solid S is the region between $x = y^2$ and $x = 1$. Cross-sections of S perpendicular to the the y-axis are squares. Find the volume of S.





$A(y) = (1-y^2)^2 = 1-2y^2+y^4$

$V = \int_{-1}^1 A(y)\,dy = 2\int_0^1 A(y)\,dy$

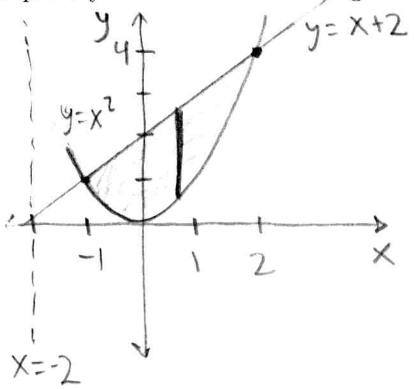
$= 2\int_0^1 1-2y^2+y^4\,dy$

$= 2\left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5\right]_0^1$

$= 2\left[1 - \frac{2}{3} + \frac{1}{5}\right]$

$= 2\left[\frac{15-10+3}{15}\right] = \boxed{\frac{16}{15}}$

3. Find the volume of the solid formed by rotating about the the line $x = -2$ the region enclosed by $y = x^2$ and $y = x + 2$. Express your answer as an integral but **do not evaluate the integral.**



Shells:

$$V = \int_{-1}^{2} 2\pi (x+2)(x+2-x^2)\, dx$$

Disks: (not recommended)

$$V = \int_0^1 \pi\left[(2+\sqrt{y})^2 - (2-\sqrt{y})^2\right] dy$$

$$+ \int_1^4 \pi\left[(2+\sqrt{y})^2 - (2+y-2)^2\right] dy$$

Intersections: $x^2 = x+2$

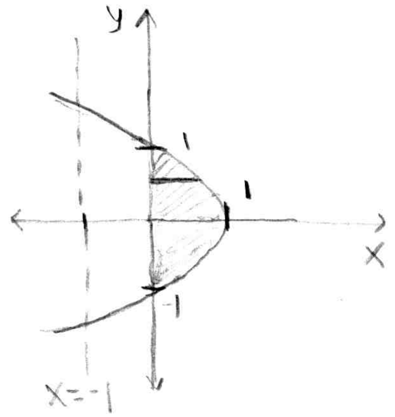
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$(2,4)$ $(-1,1)$

4. Find the volume of the solid formed by rotating about the line $x = -1$ the region enclosed by $x = 1 - y^2$ and the y-axis. Express your answer as an integral but **do not evaluate the integral.**



Disks:

$$V = \int_{-1}^{1} \pi\left[(1+1-y^2)^2 - 1\right] dy$$

Shells:

$$V = \int_0^1 2\pi (1+x)\left(2\sqrt{1-x}\right) dx$$

5. Calculate the arc length of the curve $y = \frac{2}{3}x^{\frac{3}{2}} + 4$ for $0 \le x \le 3$.

$$\frac{dy}{dx} = \frac{2}{3}\left(\frac{3}{2}\right)x^{\frac{1}{2}} = x^{\frac{1}{2}}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+(x^{1/2})^2 = 1+x.$$

Arc length: $L = \int_0^3 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^3$

$$= \frac{2}{3}\left[(1+3)^{3/2} - (1+0)^{3/2}\right]$$

$$= \frac{2}{3}\left[8-1\right]$$

$$= \boxed{\frac{14}{3}}$$

6. Find the x-coordinate of the centroid of the region enclosed by $y = 2x - x^2$ and the x-axis. It may be useful to know that the area of the region is $\frac{4}{3}$.

By symmetry: $y = 2x - x^2 = -(x-1)^2 + 1$ an upside down parabola shifted up 1 and right 1. The region is symmetrical about the line $x=1$ and therefore $\bar{x} = 1$.

The other way:

$$\bar{x} = \frac{3}{4}\int_0^2 x(2x-x^2)\,dx = \frac{3}{4}\int_0^2 2x^2 - x^3\,dx = \frac{3}{4}\left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^2$$

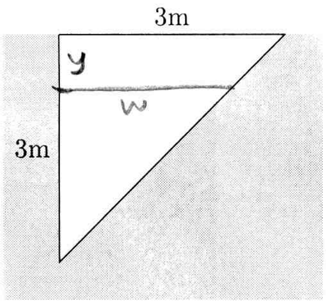
$$= \frac{3}{4}\left[\frac{16}{3} - 4\right]$$

$$= \frac{3}{4}\left[\frac{16-12}{3}\right]$$

$$= 1$$

7. Calculate the hydrostatic force against the triangle shown when it is submerged in water so that it's top edge is at the surface.

Let y = depth below surface.



$$F=\int_0^3 1000(9.8)y(3-y)\,dy = 9800\int_0^3 3y-y^2\,dy$$

$$=9800\left[\frac{3}{2}y^2-\frac{1}{3}y^3\right]_0^3$$

$$=9800\left[\frac{27}{2}-9\right]$$

$$=9800\left[\frac{27-18}{2}\right]$$

$$=9800\left(\frac{9}{2}\right)$$

$$=4900(9)$$

$$=44100 \text{ N}$$

$$\frac{w}{3-y}=\frac{3}{3}$$

$$w=3-y$$

8. A 500 lb wrecking ball hangs from a 40 ft chain with linear density 4 lb/ft attached to a crane. How much work is done if the crane lifts the ball 20 ft by drawing in the chain?

Let x = length of chain still hanging.

Force of chain: $4x$ lb

Force of ball: 500 lb

$$W=\int_{20}^{40} 500+4x\,dx = 500x+2x^2\Big|_{20}^{40} = 500(40)+2(40)^2-500(20)-2(20)^2$$

$$=500(20)+8(20)^2-2(20)^2$$

$$=10000+6(20)^2$$

$$=10000+2400$$

$$=12400 \text{ ft}\cdot\text{lb}.$$

Alternative: Let x = length of chain drawn in.

$$W=\int_0^{20} 500+4(40-x)\,dx$$