258 F12 Exam 1 Review (Chapter 5) 9/19

§1 Inverse Functions:

- One-to-one functions
- Horizontal line test
- $f^{-1}(x)=y \Leftrightarrow x=f(y)$ ← definition
- $\left.\begin{array}{l} f(f^{-1}(x))=x \\ f^{-1}(f(x))=x \end{array}\right\}$ Cancellation equations
- Method for finding the inverse of a one-to-one function:

  $y=f(x)$

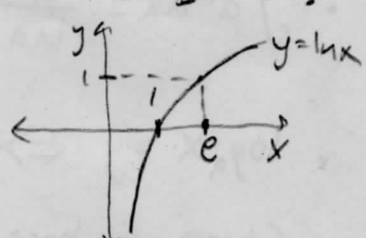
  Solve for x.
- Reflecting through the line $y=x$.
- $(f^{-1})'(a)=\dfrac{1}{f'(f^{-1}(a))}$

Problems: §1: 16, 22, 24, 30, <u>32, 34</u>

§2 The Natural Logarithm Function

- Definition: $\ln x=\int_1^x \frac{1}{t}\,dt$ for $x>0$
- $\frac{d}{dx}\ln x=\frac{1}{x}$ and $\int \frac{1}{x}\,dx=\ln|x|+C$
- Laws of logarithms

  $\left.\begin{array}{l} \ln(xy)=\ln x+\ln y \\ \ln\left(\frac{x}{y}\right)=\ln x-\ln y \\ \ln(x^r)=r\ln x \end{array}\right\}$ Same for logarithms with any base.

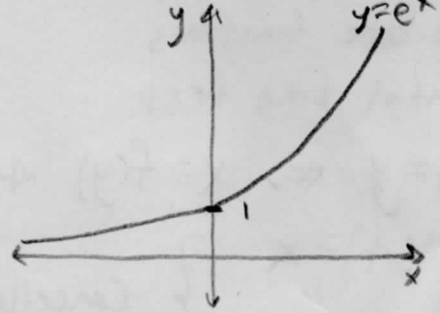


- $\lim_{x\to\infty}\ln x=\infty$ and $\lim_{x\to 0^+}\ln x=-\infty$
- Logarithmic differentiation

Problems: §2: 4, 6, 14-18, 24, 28, 30, 32, 34, <u>38</u>, 52, 54, 56, 58, 60, 62

§3. The Natural Exponential Function.

- $e^x = y \Leftrightarrow x = \ln y$ (Inverse of ln)
- $e^{\ln x} = x$ for $x > 0$
- $\ln(e^x) = x$ all x
- $\lim_{x \to -\infty} e^x = 0$ $\lim_{x \to \infty} e^x = \infty$



- Laws of exponents

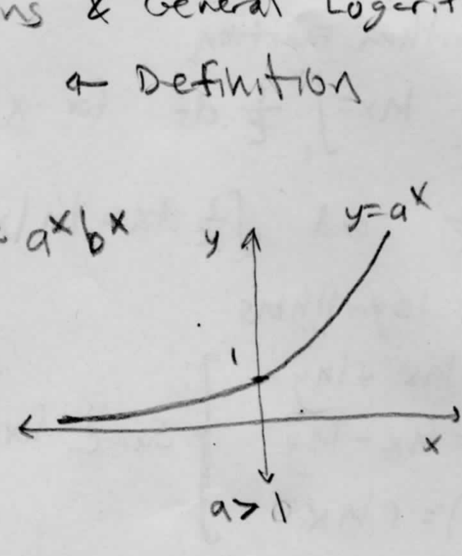
$$\left.\begin{array}{l} e^{x+y} = e^x e^y \\ e^{x-y} = \dfrac{e^x}{e^y} \\ (e^x)^r = e^{rx} \end{array}\right\} \text{same for exponents with any base}$$

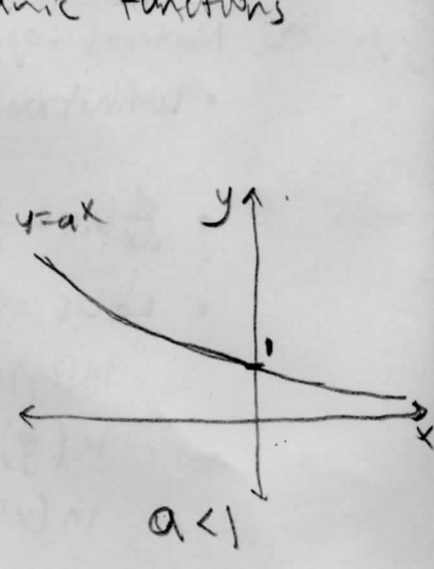
- $\frac{d}{dx} e^x = e^x$ and $\int e^x \, dx = e^x + C$

Problems: §3: 2-10, 12, 22, 26, 30, 36, 42, 48, 58-62

§4 General Exponential Functions & General Logarithmic Functions

- $a^x = e^{x \ln a}$ for $a > 0$ ← Definition
- Laws of exponents
  As above + $(ab)^x = a^x b^x$
- $\frac{d}{dx} a^x = a^x \ln a$
- $\int a^x \, dx = \frac{a^x}{\ln a} + C$





- $\log_a x = y \Leftrightarrow x = a^y$ ← Definition as inverse of a^x
- Change of base: $\log_a x = \frac{\ln x}{\ln a}$
- $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$
- $e = \lim_{x \to 0} (1+x)^{1/x} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

Problems: §4; 4, 6, 8a, 9b, 10, 22, 24-38, 42-46

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§5 Exponential Growth and Decay

- Law of natural growth $\frac{dy}{dt} = ky$
- Its solution $y(t) = y(0)e^{kt}$
- Solving for $y(0)$, k, t.

Problems: §5: 2, 4, 8, 12, 14, <u>20</u>.

§6 Inverse Trig Functions

- $\sin^{-1}x = y \iff x = \sin y$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

  Cancellation equations:

  $\sin^{-1}(\sin x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

  $\sin(\sin^{-1}x) = x$ for $-1 \le x \le 1$

  $$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

- $\cos^{-1}x = y \iff x = \cos y$ and $0 \le y \le \pi$

  Cancellation equations;

  $\cos^{-1}(\cos x) = x$ for $0 \le x \le \pi$

  $\cos(\cos^{-1}x) = x$ for $-1 \le x \le 1$

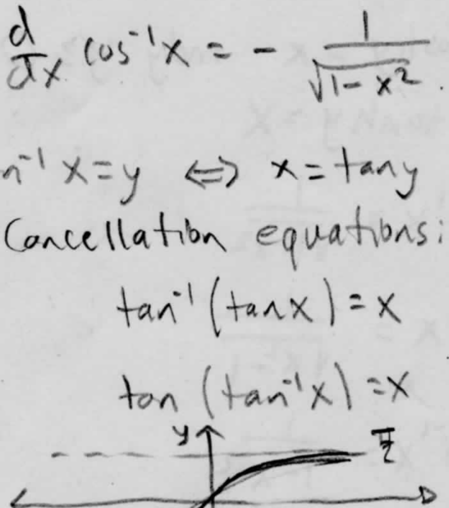
  $$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}.$$

- $\tan^{-1}x = y \iff x = \tan y$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

  Cancellation equations:

  $\tan^{-1}(\tan x) = x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

  $\tan(\tan^{-1}x) = x$ for all x



$y = \tan^{-1}x$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

- Know other inverse trig functions exist.
  No derivatives needed.
  No need to memorize ranges.

Problems §6: 4, 16, 20, 30, 32

§7 Hyperbolic Functions

- $\sinh x = \dfrac{e^x - e^{-x}}{2}$ • $\operatorname{csch} x = \dfrac{1}{\sinh x}$
- $\cosh x = \dfrac{e^x + e^{-x}}{2}$ • $\operatorname{sech} x = \dfrac{1}{\cosh x}$
- $\tanh x = \dfrac{\sinh x}{\cosh x}$ • $\coth x = \dfrac{\cosh x}{\sinh x}$

- $\cosh^2 x - \sinh^2 x = 1$
  and division by $\cosh^2 x$: $1 - \tanh^2 x = \operatorname{sech}^2 x$
  (division by $\sinh^2 x$? $\coth^2 x - 1 = \operatorname{csch}^2 x$)

- Derivatives: $\dfrac{d}{dx} \sinh x = \cosh x$

  $\dfrac{d}{dx} \cosh x = \sinh x$

  $\dfrac{d}{dx} \tanh x = \operatorname{sech}^2 x$

- Inverses: $y = \sinh^{-1} x \iff \sinh y = x$

  $y = \cosh^{-1} x \iff \cosh y = x$ and $y \geq 0$

  $y = \tanh^{-1} x \iff \tanh y = x$

- Their derivatives: $\dfrac{d}{dx} \sinh^{-1} x = \dfrac{1}{\sqrt{1+x^2}}$

  $\dfrac{d}{dx} \cosh^{-1} x = \dfrac{1}{\sqrt{x^2-1}}$

  $\dfrac{d}{dx} \tanh^{-1} x = \dfrac{1}{1-x^2}$

Problems §7: 2, 4b, 8, 26-36.

§8 Indeterminate Forms and l'Hospital's Rule.

- l'Hospital's rule: If $\lim_{x\to a} \frac{f(x)}{g(x)}$ has I.F. $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.
- Indeterminate form $0\cdot\infty$ and conversion to $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
- I.F. $\infty-\infty$ and conversion to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ or $0\cdot\infty$
- I.F.s 0^0, ∞^0, 1^∞ and conversion to IF $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Problems §8: 4, 10, 18, 22, 24, 26, 32, 34.

Review Exercises:

Odds: 3-9, 11a, 13-19, 21, 23, 31-43, 49, 51, 57, 65, 67-79.

Evens correspond.