NAMES:

1. In this problem we will consider a generic function given as a power series centered at 0:

$$f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots.$$

a) Differentiate the power series and evaluate at $x = 0$ to find $f(0)$, $f'(0)$, $f''(0)$, $f^{(3)}(0)$, and $f^{(4)}(0)$ (your answers will be in terms of the constants $c_0, c_1, c_2, \dots$).

b) Find a formula relating $f^{(n)}(0)$ and c_n (your formula should involve the factorial: $n! = n(n-1)(n-2)\dots(2)1$).

c) Check that your formula works for $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ by differentiating the left hand side of the equation and evaluating at $x = 0$

2. a) Use the formula you found in 1(b) to find a power series for e^x (that is, suppose $e^x = \sum_{n=0}^{\infty} c_n x^n$ and then find $c_0,\ c_1,\ c_2,\ c_3,\ c_4$, and a formula for c_n).

b) Use the ratio test to find the radius of convergence of the power series for e^x.

3. a) Use the formula of 1(b) to find a power series for $\sin x$.

b) Differentiate the power series for $\sin x$ to find a power series for $\cos x$.