Math 258 Exam 1 September 25, 2015

INSTRUCTIONS: Answer all 12 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted. When evaluating a limit your answer should either be a number, ∞ , $-\infty$, or DNE.

FORMULAS:

DRMULAS:

$$e = \lim_{x \to 0} (1+x)^{\frac{1}{x}} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n}$$

$$\frac{d}{dx} \left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^{2}}}$$

$$\frac{d}{dx} \left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^{2}}}$$

$$\frac{d}{dx} \left(\tan^{-1} x\right) = \frac{1}{1+x^{2}}$$

$$\frac{d}{dx} \left(\csc^{-1} x\right) = -\frac{1}{x\sqrt{x^{2}-1}}$$

$$\frac{d}{dx} \left(\sec^{-1} x\right) = \frac{1}{1+x^{2}}$$

$$\frac{d}{dx} \left(\cot^{-1} x\right) = -\frac{1}{1+x^{2}}$$

$$\frac{d}{dx} \left(\coth x\right) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx} \left(\operatorname{csch} x\right) = -\operatorname{csch} x \tanh x$$

$$\frac{d}{dx} \left(\coth x\right) = -\operatorname{csch}^{2} x$$

$$\sinh^{-1}(x) = \ln\left(x + \sqrt{x^{2}+1}\right)$$

$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^{2}-1}\right)$$

$$\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$$\frac{d}{dx} \left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^{2}+1}}$$

$$\frac{d}{dx} \left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^{2}-1}}$$

$$\frac{d}{dx} \left(\tanh^{-1} x\right) = \frac{1}{1-x^{2}}$$

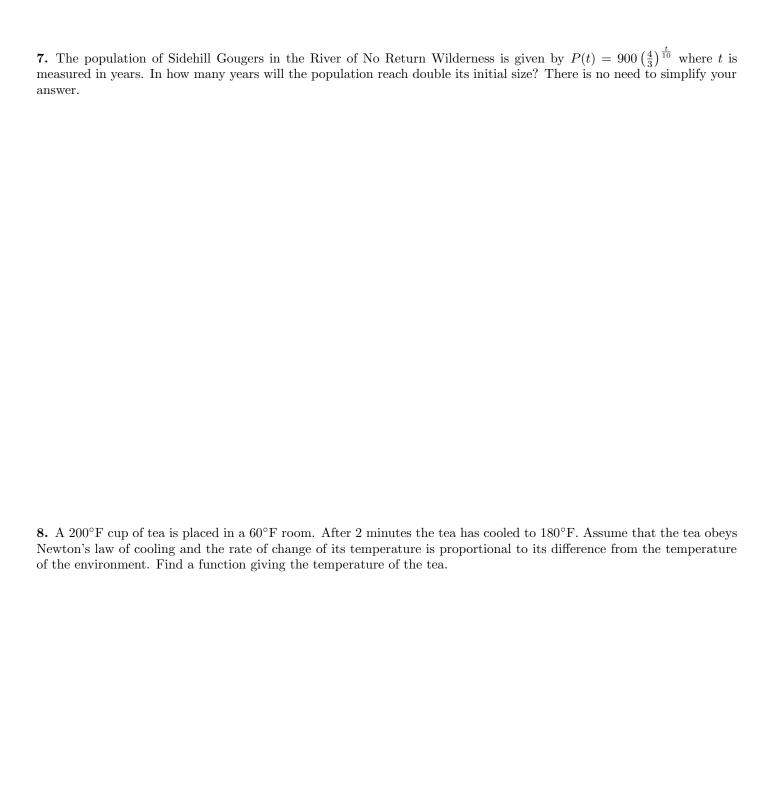
PROBLEMS:

1. Differentiate $f(x) = \ln(xe^x)$.

2. Differentiate $g(t) = (\ln t)^t$.
3. Find an equation for the line tangent to the curve $y = \tanh^{-1}(\sqrt{x})$ at the point $(\frac{1}{4}, \frac{1}{2}\ln(3))$.
4. Evaluate the expression $\sin(\tan^{-1} 5)$

5. Evaluate the integral $\int e^x \sin(e^x) dx$.

6. Evaluate the integral $\int_4^9 \frac{2^{\sqrt{t}}}{\sqrt{t}} dt$.



9. Find the limit $\lim_{x \to 1} \frac{\ln x}{1 - x^2}$.

10. Find the limit $\lim_{x \to \infty} [x \tan^{-1} x]$.

11. Find the limit $\lim_{t\to\infty} \left[\sqrt{t^2 + 3t} - t \right]$.

12. Find the limit $\lim_{x\to\infty} x^{\frac{1}{x}}$.