

NAME:

MATH 258

EXAM 1

SEPTEMBER 25, 2015

INSTRUCTIONS: Answer all 12 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted. When evaluating a limit your answer should either be a number,  $\infty$ ,  $-\infty$ , or DNE.

FORMULAS:

$$e = \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} (\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} (\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} (\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx} (\coth x) = -\operatorname{csch}^2 x$$

$$\sinh^{-1}(x) = \ln \left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1}(x) = \ln \left(x + \sqrt{x^2 - 1}\right)$$

$$\tanh^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x}\right)$$

$$\frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}$$

$$\frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$$

$$\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2}$$

PROBLEMS:

1. Differentiate  $f(x) = \ln(xe^x)$ .

2. Differentiate  $g(t) = (\ln t)^t$ .

3. Find an equation for the line tangent to the curve  $y = \tanh^{-1}(\sqrt{x})$  at the point  $(\frac{1}{4}, \frac{1}{2} \ln(3))$ .

4. Evaluate the expression  $\sin(\tan^{-1} 5)$

**5.** Evaluate the integral  $\int e^x \sin(e^x) dx$ .

**6.** Evaluate the integral  $\int_4^9 \frac{2^{\sqrt{t}}}{\sqrt{t}} dt$ .

**7.** The population of Sidehill Gougiers in the River of No Return Wilderness is given by  $P(t) = 900\left(\frac{4}{3}\right)^{\frac{t}{10}}$  where  $t$  is measured in years. In how many years will the population reach double its initial size? There is no need to simplify your answer.

**8.** A 200°F cup of tea is placed in a 60°F room. After 2 minutes the tea has cooled to 180°F. Assume that the tea obeys Newton's law of cooling and the rate of change of its temperature is proportional to its difference from the temperature of the environment. Find a function giving the temperature of the tea.

**9.** Find the limit  $\lim_{x \rightarrow 1} \frac{\ln x}{1 - x^2}$ .

**10.** Find the limit  $\lim_{x \rightarrow \infty} [x \tan^{-1} x]$ .

**11.** Find the limit  $\lim_{t \rightarrow \infty} \left[ \sqrt{t^2 + 3t} - t \right]$ .

**12.** Find the limit  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$ .