NAME:

MATH 258 EXAM 3 NOVEMBER 4, 2015

INSTRUCTIONS: Answer all 8 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad \cot^2 u + 1 = \csc^2 u$$

$$\int \tan u \; du = \ln|\sec u| + C \qquad \int \sec u \; du = \ln|\sec u + \tan u| + C \qquad \int \frac{1}{u^2 + a^2} \; du = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$V = \int_a^b A(x) \; dx \qquad V = \int_a^b \pi[r(x)]^2 \; dx \qquad V = \int_a^b 2\pi r(x) f(x) \; dx$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad SA = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} \; dx \qquad W = \int_a^b f(x) \; dx$$

$$P = \frac{F}{A} = \rho g d \qquad \overline{x} = \frac{1}{A} \int_a^b x f(x) \; dx \qquad \overline{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2 \; dx$$

1. Determine the area enclosed between the curves $x + y = 2$ and $x = y^2$.

2. Determine the volume of the solid formed when the region enclosed between $y = x^2$ and $y = 2x$ is rotated around the y-axis.

3. The base of a solid is the region bounded by the parabolas $y = 1 - x^2$ and $y = (x-1)^2$ and cross-sections perpendicular to the x-axis are squares. Set up but *do not evaluate* an integral giving the volume of the solid.

4. Calculate the length of the curve $y = \ln(\cos x)$ for $0 \leq x \leq \dfrac{\pi}{4}$.

5. Set up but *do not evaluate* an integral giving the area of the surface formed by rotating the curve $y = \frac{1}{x}$, $x \geq 1$, about the x-axis.

6. A 4-lb bucket of candy is attached to the end of a rope that weighs 0.5 lb/ft. How much work is required to raise the bucket from the ground to the treehouse, a height of 20 ft? Make sure to indicate the units of measurement for your answer.

7. Find the centroid of the region enclosed between $y = 1 - x^4$ and the x-axis. Hint: use symmetry.

8. A vertical plate with the indicated shape is submerged in water as shown. Set up but *do not evaluate* an integral giving the hydrostatic force on one side of the plate. Recall that water has a density of 1000 kg/m^3 and the acceleration of gravity is 9.8 m/s^2.