Name:

Math 258 — Exam 4 — December 4, 2015

Instructions: Answer all 9 problems. Calculators, notes, cell phones, and other materials are not permitted. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. When the solution to a problem is a power series and you are not given other instructions, write out the first 4 non-zero terms of the series or use sigma notation to describe the series. For example, $\tan^{-1}(2x) = 2x - \frac{8}{3}x^3 + \frac{32}{5}x^5 - \frac{128}{7}x^7 + \dots$ or $\tan^{-1}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{2n+1} x^{2n+1}$.

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \dots$	$R = 1$

1. Determine whether the sequence $\left\{ \frac{2\sqrt{n}}{1+n} \right\}$ converges or diverges. If it converges, find the limit.

2. Determine whether the series $\sum_{n=0}^{\infty} \frac{1+3^n}{4^n}$ is convergent or divergent. If it is convergent, find its sum.

3. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$ is convergent or divergent.

4. Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ is convergent or divergent.

5. The n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = a_1 + a_2 + \cdots + a_n$. Suppose $\lim_{n\to\infty} s_n = 12042015$. Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Explain why or why not.

6. Find the Taylor series for $f(x) = x^4$ centered at $a = 1$. Give all non-zero terms of the series.

7. Calculate the coefficient of x^{12} in the Maclaurin series for $f(x) = \cos(2x^3)$. Hint: use the Maclaurin series we have already found.

8. Evaluate the indefinite integral as a series: $\int \frac{e^x - 1}{x} dx$

9. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}(x-2)^n$. Note that this power series is centered at $a = 2$.