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|---|---|---|
| $\dfrac{d\sin^{-1}x}{dx}=\dfrac{1}{\sqrt{1-x^2}}$ | $\dfrac{d\cos^{-1}x}{dx}=-\dfrac{1}{\sqrt{1-x^2}}$ | $\dfrac{d\tan^{-1}x}{dx}=\dfrac{1}{1+x^2}$ |
| $\dfrac{d\csc^{-1}x}{dx}=-\dfrac{1}{x\sqrt{x^2-1}}$ | $\dfrac{d\sec^{-1}x}{dx}=\dfrac{1}{x\sqrt{x^2-1}}$ | $\dfrac{d\cot^{-1}x}{dx}=-\dfrac{1}{1+x^2}$ |
| $\dfrac{d\ \mathrm{csch}x}{dx}=-\ \mathrm{csch}x\coth x$ | $\dfrac{d\ \mathrm{sech}x}{dx}=-\ \mathrm{sech}x\tanh x$ | $\dfrac{d\coth x}{dx}=-\ \mathrm{csch}^2x$ |
| $\sinh^{-1}(x)=\ln\left(x+\sqrt{x^2+1}\right)$ | $\cosh^{-1}(x)=\ln\left(x+\sqrt{x^2-1}\right)$ | $\tanh^{-1}(x)=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ |
| $\dfrac{d\sinh^{-1}x}{dx}=\dfrac{1}{\sqrt{x^2+1}}$ | $\dfrac{d\cosh^{-1}x}{dx}=\dfrac{1}{\sqrt{x^2-1}}$ | $\dfrac{d\tanh^{-1}x}{dx}=\dfrac{1}{1-x^2}$ |
| $\sin^2u=\dfrac{1}{2}(1-\cos 2u)$ | $\cos^2u=\dfrac{1}{2}(1+\cos 2u)$ | $\sin 2u=2\sin u\cos u$ |
| $e=\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n$ | $1+\tan^2u=\sec^2u$ | $\cot^2u+1=\csc^2u$ |
| $\int\tan u\ du=\ln\lvert\sec u\rvert+C$ | $\int\sec u\ du=\ln\lvert\sec u+\tan u\rvert+C$ | $\int\dfrac{1}{u^2+a^2}\ du=\dfrac{1}{a}\tan^{-1}\left(\dfrac{u}{a}\right)+C$ |
| $V=\int_a^b A(x)\ dx$ | $V=\int_a^b \pi[r(x)]^2\ dx$ | $V=\int_a^b 2\pi r(x)f(x)\ dx$ |
| $ds=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ | $SA=\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\ dx$ | $W=\int_a^b f(x)\ dx$ |
| $P=\dfrac{F}{A}=\rho gd$ | $\overline{x}=\dfrac{1}{A}\int_a^b xf(x)\ dx$ | $\overline{y}=\dfrac{1}{A}\int_a^b \dfrac{1}{2}\left[f(x)\right]^2\ dx$ |

**1.** Find a formula for $f^{-1}(x)$ if $f(x)=\dfrac{4}{2x+1}$.

**2.** Calculate $\left(f^{-1}\right)'(2)$ for $f(x)=2x+\ln x$.

**3.** Evaluate the improper integral or show that it diverges. $\displaystyle\int_0^2\frac{1}{(2-x)^2}dx$

**4.** A bacteria culture grows at a rate proportional to its size. After 2 hours there are 90 bacteria and after 4 hours there are 810 bacteria in the culture. Find the size of the initial population of bacteria in the culture.

**5.** Differentiate $y=\ln\left(\dfrac{1}{x}\right)+\dfrac{1}{\ln x}$

**6.** Evaluate the integral $\displaystyle\int\frac{1}{t^2-4t-12}dt$

**7.** Evaluate the integral $\displaystyle\int_0^1\frac{\sqrt{\tan^{-1}x}}{1+x^2}dx$

**8.** Evaluate the improper integral or show that it diverges. $\displaystyle\int_1^e\frac{1}{x(\ln x)^2}dx$

**9.** The height of a pyramid is 10 meters. Horizontal cross-sections $x$ meters from the top are rectangles with side lengths $x$ and $2x$. What is the volume of the pyramid?

**10.** Evaluate the integral $\int t\sin(2t)\ dt$.

**11.** Evaluate the integral $\int \tan^4\theta\ d\theta$.

**12.** Evaluate the integral $\int_0^3 x\sqrt{9-x^2}dx$

**13.** Evaluate the integral $\int \frac{1}{t^2\sqrt{t^2-1}}dt$.

**14.** Evaluate the integral $\int \frac{x^2}{x+4}dx$.

**15.** Evaluate the integral $\int \frac{4}{x(x-2)}dx$.

**16.** Evaluate the integral $\int \frac{x-2}{x(x^2+1)}dx$

**17.** Find the limit $\lim_{x\to 0}\frac{\sinh x}{x}$.

**18.** Calculate the area of the region enclosed by the curves $y=x^2$ and $y=2x-x^2$.

**19.** Calculate the volume of the solid obtained by rotating the region between $x=y^2$ and $x=1$ about the $y$-axis.

**20.** Set up but **do not evaluate** an integral giving the volume of the solid obtained by rotating the region between the curves $y=\sin x$ and $y=\cos x$ for $-\frac{3\pi}{4}\le x\le\frac{\pi}{4}$ about the line $x=\frac{\pi}{2}$ .

**21.** Calculate the arc length of the curve $y=\cosh x$ for $0\le x\le \ln 5$. You may want to make use of the identity $\cosh^2 x = 1+\sinh^2 x$.

**22.** Find the $y$-coordinate of the centroid of the region bounded by $y=e^x$, the $x$-axis, the $y$-axis, and $x=\ln 4$. It may be helpful to know that the area of the region is 3.

**23.** A flexible, 120 ft rope with a total weight of 40 lb lies coiled at the base of a cliff. One end is tied to a rock climber who climbs to a height of 60 ft. How much work did the climber do in raising her end of the rope?

**24.** Determine if the sequence $\left\{\sqrt{4-\frac{1}{n}}\right\}_{n=1}^{\infty}$ converges or diverges. If it converges, find its limit.

**25.** Determine whether the series $\sum_{n=0}^{\infty}\frac{(-2)^{n+1}}{3^n}$ is convergent or divergent. If it is convergent, find its sum.

**26.** Determine whether the series $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+3}$ is convergent or divergent.

**27.** Determine whether the series $\sum_{n=4}^{\infty}(-1)^n 2^{-n^2}$ is absolutely convergent, conditionally convergent, or divergent.

**28.** Find a power series representation of the function $f(x)=\frac{1}{(2-x)^2}$ and determine its interval of convergence.

**29.** Find a series representation of $\frac{1}{x}\sin x$ and use this series to evaluate the integral $\int \frac{1}{x}\sin x\ dx$ as a series. Either write your answer using a $\sum$ or find the first 4 terms of the series. Do not simplify your answer.

**30.** Find the Taylor series for $f(x)=\frac{1}{x}$ at $a=-1$. Either write your answer using a $\sum$ or find the first 5 terms of the series.