Directly. Possibly after some algebra (multiplication or division, generally).

a) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$

b) $\displaystyle\int (x-1)(x+2)\,dx$

c) $\displaystyle\int e^{1+\ln x}\,dx$

Substitution. $\displaystyle\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$. Watch for compositions of functions (you may need to do algebra here too).

a) $\displaystyle\int \frac{x}{4+x}\,dx$

b) $\displaystyle\int \frac{1}{x\ln x}\,dx$

c) $\displaystyle\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$

d) $\displaystyle\int (1+3x)^7\,dx$

e) $\displaystyle\int x\ln(x-1) + x\ln(x+1)\,dx$

Partial Fractions. Watch for rational functions $\dfrac{P(x)}{Q(x)}$ (where $P(x)$ and $Q(x)$ are polynomials). Division first: the degree of $P(x)$ must be lower than the degree of $Q(x)$. Different forms are based on the factorization of the denominator (cases I–III are the most important).

a) $\displaystyle\int \frac{1}{x^2+3x-4}\,dx$

b) $\displaystyle\int \frac{x^3-4x-10}{x^2-x-6}\,dx$

c) $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$

d) $\displaystyle\int \frac{x^2}{(x-3)(x+2)^2}\,dx$

Trig substitution. Three expressions to watch for:

1. $\sqrt{a^2-x^2}$ substitute $x = a\sin\theta$, identity $1-\sin^2\theta = \cos^2\theta$;
2. $\sqrt{a^2+x^2}$ substitute $x = a\tan\theta$, identity $1+\tan^2\theta = \sec^2\theta$;
3. $\sqrt{x^2-a^2}$ substitute $x = a\sec\theta$, identity $\sec^2\theta - 1 = \tan^2\theta$;

Converts the problem into a trig integral (see below). Use trig substitution only when regular substitution won't work.

a) $\displaystyle\int \frac{x^3}{\sqrt{16-x^2}}\,dx$

b) $\displaystyle\int x^3\sqrt{x^4+4}\,dx$

c) $\displaystyle\int \sqrt{1-9x^2}\,dx$

d) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-9}}$

Trig integral. An integral involving only trig functions should be approached using the techniques outlined in the margins of section 6.2.

a) $\int \sin^6 x \cos^3 x \, dx$

b) $\int \sin^2(2x) \, dx$

c) $\int \tan^4 x \, dx$

d) $\int \sec x \tan^3 x \, dx$

Integration by Parts. $\int u \, dv = uv - \int v \, du$. Watch for products of functions.

a) $\int xe^{-x} \, dx$

b) $\int x^2 \cos x \, dx$

c) $\int \sqrt{x} \ln x \, dx$

d) $\int (x^2 + 1) \cos x \, dx$

e) $\int \ln x \, dx$

Power Series. If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ for $|x - a| < R$, then $\int f(x) \, dx = C + \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1}$ for $|x - a| < R$. Usually the last resort.

a) $\int e^{-x^2} \, dx$

b) $\int \frac{1}{2 - x^4} \, dx$

c) $\int (1 + x^2)^{\frac{3}{4}} \, dx$

Geometrically. Definite integrals correspond to areas under curves. You may be able to to use geometry to calculate the areas.

a) $\int_0^2 \sqrt{4 - x^2} \, dx$

b) $\int_1^5 5 - x \, dx$

c) $\int_1^2 \sqrt{2y - y^2} \, dy$