NAME(S): Solutions
MATH 258 GENERAL EXPONENTIAL AND LOGARITHMIC FUNCTIONS SEPTEMBER 4, 2015

1. Evaluate the expressions,

(a) $\log_8 320 - \log_8 5 = \log_8\left(\frac{320}{5}\right) = \log_8 64 = 2$

(b) $4^{\log_2 6} = (2^2)^{\log_2 6} = 2^{2\log_2 6} = (2^{\log_2 6})^2 = 6^2 = 36$

2. Order the following numbers from least to greatest. I think you should be able to do this without a calculator; if it helps $e \approx 2.71828$ and $\sqrt{10} \approx 3.16278$.

(a) $5^{\sqrt{10}} \approx 5^{3.16278}$

(b) $(5^{10})^{\frac{1}{2}} = 5^5$

$e \le c \le d \le a \le b$

(c) $\ln(5^{10}) = 10 \ln 5$

(d) $\log_2(5^{10}) = 10 \log_2 5$

(e) $\log_{10}(5^{10}) = 10 \log_{10} 5$

3. Differentiate the following functions.

(a) $f(\theta) = 2^{\cos\theta}$

$$f'(\theta) = \frac{d}{d\theta}\left[e^{(\ln 2)\cos\theta}\right] = e^{(\ln 2)\cos\theta}\,\frac{d}{d\theta}\left[(\ln 2)\cos\theta\right]$$

$$= 2^{\cos\theta}(\ln 2)(-\sin\theta)$$

(b) $g(x) = x^{\frac{1}{x}}$

$$g'(x) = \frac{d}{dx}\left[e^{\frac{\ln x}{x}}\right] = e^{\frac{\ln x}{x}}\,\frac{d}{dx}\left[\frac{\ln x}{x}\right]$$

$$= x^{\frac{1}{x}}\left[\frac{1-\ln x}{x^2}\right]$$

4. Evaluate the following integrals.

(a) $\int x^3 + 3^x\,dx \;=\; \frac{1}{4}x^4 + \frac{3^x}{\ln 3} + C$

(b) $\int_0^1 \frac{2^t}{1+2^t}dt \;=\; \int_2^3 \frac{1}{\ln 2\, u}\,du$ with the substitution $u = 1+2^t$ $\;\frac{1}{\ln 2}du = 2^t dt$

$$= \frac{1}{\ln 2}\ln|u|\,\Big|_2^3$$

$$= \frac{1}{\ln 2}\left[\ln 3 - \ln 2\right] = \frac{\ln 3}{\ln 2} - 1$$