NAME(S): Solutions
MATH 258 INDETERMINATE FORMS AND L'HOSPITAL'S RULE SEPTEMBER 15, 2015

Theorem (l'Hospital's rule). *If* $\lim_{x\to a} \frac{f(x)}{g(x)}$ *has indeterminate form* $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$, *then* $\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$

1. Evaluate the limits (answer either a number, $-\infty$, ∞, or DNE).

(a) $\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$

$\frac{\infty}{\infty}$ I.F. Apply l'Hospital's rule.

(b) $\lim_{x\to 1} \frac{\ln x}{2x-2} = \lim_{x\to 1} \frac{1/x}{2} = \frac{1}{2}$

$\frac{0}{0}$ I.F. Apply l'Hospital's rule

(c) $\lim_{x\to 0+} \frac{\ln x}{x} = -\infty$ (because x approaches 0 from the right)

$\frac{-\infty}{0}$ not indeterminate

2. Evaluate the limits (answer either a number, $-\infty$, ∞, or DNE).

(a) $\lim_{x\to\infty} \frac{\sinh x}{x} = \lim_{x\to\infty} \frac{\cosh x}{1} = \infty$

$\frac{\infty}{\infty}$ I.F.

(b) $\lim_{x\to 0} \frac{\sinh x}{x} = \lim_{x\to 0} \frac{\cosh x}{1} = 1$

$\frac{0}{0}$ I.F.

(c) $\lim_{x\to 0} \frac{\cosh x}{x^2} = \infty$ (because x^2 is always positive)

$\frac{1}{0}$ not indeterminate

(d) $\lim_{x\to\infty} \frac{\cosh x}{x^2} = \lim_{x\to\infty} \frac{\sinh x}{2x} = \lim_{x\to\infty} \frac{\cosh x}{2} = \infty$

$\frac{\infty}{\infty}$ I.F. Still $\frac{\infty}{\infty}$ I.F.

(e) $\lim_{x\to -\infty} \frac{\tanh x}{x} = 0$

$\frac{-1}{-\infty}$ not indeterminate

3. Evaluate the limits (answer either a number, $-\infty$, ∞, or DNE).

(a) $\lim_{x\to-\infty} x^2e^x$. Hint: $x^2e^x = \frac{?}{?}$.

$$\lim_{x\to-\infty} x^2e^x = \lim_{x\to-\infty} \frac{x^2}{e^{-x}} = \lim_{x\to-\infty} \frac{2x}{-e^{-x}} = \lim_{x\to-\infty} \frac{2}{e^{-x}} = 0$$

$\frac{\infty}{\infty}$ I.F. still $\frac{\infty}{\infty}$ I.F.

(b) $\lim_{x\to 0+} x\ln x$. Hint: $x\ln x = \frac{\ln x}{?}$.

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = 0$$

$\frac{\infty}{\infty}$ I.F.

(c) $\lim_{x\to 0+} x^x$. Hint: $x^x = e^?$.

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = e^{\lim_{x\to 0^+}(x\ln x)}$$ because the exponential function is continuous

$= e^0$ using part b

$= 1$