NAME(S):

MATH 258 — SEQUENCES — NOVEMBER 6, 2015

Definiton. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence and L be a number. Then $\lim\limits_{n\to\infty} a_n = L$ if and only if for every number $\epsilon > 0$ there is a number N such that if $n > N$, then $|a_n - L| < \epsilon$.

1. This problem deals with the sequence $\left\{e^{-n}\right\}$.

a) Find the number L such that $L = \lim_{n\to\infty} e^{-n}$.

b) If your answer for part a is correct, then there should be a number N such that if $n > N$, then $|e^{-n} - L| < \frac{1}{10}$. Find the smallest N that will work.

c) Prove using Definition 1 that your limit is correct (this means finding a formula giving N as a function of ϵ so that if $n > N$, then $|e^{-n} - L| < \epsilon$).

2. The negation of Definition 1 is: $\lim\limits_{n\to\infty} a_n \neq L$ if and only if there is some number $\epsilon > 0$ such that for every number N there is a number $n > N$ with $|a_n - L| \geq \epsilon$. Use this to prove that $\lim\limits_{n\to\infty} \sin\left(\dfrac{\pi}{6}n\right) \neq 0$ (this means finding a number $\epsilon > 0$ and a way of finding a number n such that $n > N$ and $\left|\sin\left(\frac{\pi}{6}\right)\right| \geq \epsilon$).

3. This problem concerns sums of the terms of the sequence $\left\{\dfrac{1}{2^n}\right\}_{n=1}^{\infty}$.

a) Write out the first 6 terms of the sequence $\left\{\frac{1}{2^n}\right\}$.

b) Let s_n be the sum of the first n terms of the sequence $\left\{\frac{1}{2^n}\right\}$ (so $s_1 = \frac{1}{2}$, $s_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, and so on). Write out s_3, s_4, s_5, s_6,... until you see a pattern. Use this to give a formula for s_n.

c) You now understand the sequence $\{s_n\}_{n=1}^{\infty}$. Calculate $\lim_{n\to\infty} s_n$.