

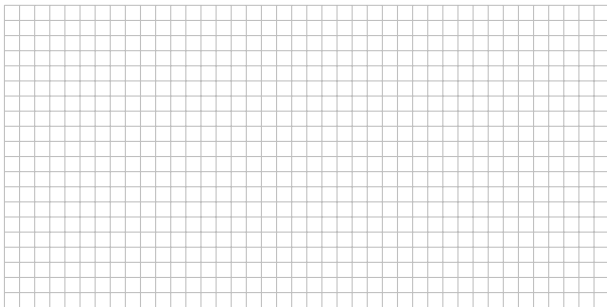
1. Determine if the series converges or diverges.

a) 
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$$

b) 
$$\sum_{n=1}^{\infty} \frac{1 + \cos n}{2^n}$$

2. We know that the harmonic series  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges, but how quickly does it diverge?

a) Use the graph of  $y = \frac{1}{x}$  and rectangles representing the series  $\sum_{n=1}^{\infty} \frac{1}{n}$  to show that if  $s_n$  is the  $n^{\text{th}}$  partial sum of the harmonic series, then  $s_n \leq 1 + \ln n$ .



b) Use part a and a calculator to find upper bounds for the one millionth partial sum  $s_{1,000,000}$  and the one billionth partial sum  $s_{1,000,000,000}$ . Observe that the harmonic series diverges very slowly.

**Theorem** (The Ratio Test). Suppose that  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ .

i) If  $L < 1$ , then the series  $\sum_{n=1}^{\infty} a_n$  is absolutely convergent.

ii) If  $L > 1$ , then the series  $\sum_{n=1}^{\infty} a_n$  is divergent.

iii) If  $L = 1$ , then the test is inconclusive.

**Theorem** (The Root Test). Suppose that  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$ .

i) If  $L < 1$ , then the series  $\sum_{n=1}^{\infty} a_n$  is absolutely convergent.

ii) If  $L > 1$ , then the series  $\sum_{n=1}^{\infty} a_n$  is divergent.

iii) If  $L = 1$ , then the test is inconclusive.

**3.** Use the ratio or the root test to determine if each series is convergent or divergent.

a)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

b)  $\sum_{n=1}^{\infty} \frac{(-3)^n}{2^n n^2}$

c)  $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

d)  $\sum_{n=1}^{\infty} \left( \frac{-2n}{n+1} \right)^{5n}$