Names:

Math 258 Series November 17, 2015

1. Determine if the series converges or diverges.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\cos n}{2^n}$

2. We know that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, but how quickly does it diverge?

a) Use the graph of $y = \frac{1}{x}$ and rectangles representing the series $\sum_{n=1}^{\infty} \frac{1}{n}$ to show that if s_n is the n^{th} partial sum of the harmonic series, then $s_n \le 1 + \ln n$.

b) Use part a and a calculator to find a upper bounds for the one millionth partial sum $s_{1,000,000}$ and the one billionth partial sum $s_{1,000,000,000}$. Observe that the harmonic series diverges very slowly.

Theorem (The Ratio Test). *Suppose that* $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$.

i) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

ii) If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

iii) If $L = 1$, then the test is inconclusive.

Theorem (The Root Test). *Suppose that* $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$.

i) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

ii) If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

iii) If $L = 1$, then the test is inconclusive.

3. Use the ratio or the root test to determine if each series is convergent or divergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{2^n n^2}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{100^n}$

d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$