$$\boxed{\frac{1}{1-u} = \sum_{n=0}^{\infty} u^n \text{ if } |u| < 1}$$

1. Use the formula above to express $\dfrac{3}{1-2x}$ as a power series. What is the radius of convergence of the power series?

2. The goal of this problem is to find two different power series representations of $\dfrac{1}{2-x}$.

a) Use the expression $\dfrac{1}{2-x} = \left(\dfrac{1}{2}\right)\left(\dfrac{1}{1-\frac{x}{2}}\right)$ and the formula at the top of the page to obtain a power series representation for $\dfrac{1}{2-x}$.

b) Use the expression $\dfrac{1}{2-x} = \dfrac{1}{1-(x-1)}$ to obtain a different power series representation for $\dfrac{1}{2-x}$.

c) What are the radii of convergence of the two power series?

Theorem 2 in the book says that derivatives and integrals work for power series. Consequently, if $|x| < 1$, then

$$\frac{1}{(1-x)^2} = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left(\sum_{n=0}^{\infty} x^n\right) = \frac{d}{dx}\left(1 + x + x^2 + x^3 + x^4 + \dots\right) = 1 + 2x + 3x^2 + 4x^3 + \dots = \sum_{n=1}^{\infty} n x^{n-1}$$

and

$$-\ln|1-x| + C = \int \frac{1}{1-x} dx = \int \left(\sum_{n=0}^{\infty} x^n\right) dx = \int \left(1 + x + x^2 + x^3 + \dots\right) dx = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

for some value of C (and it turns out that $C = 0$).

3. We know that if $|x| < 1$, then $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \dots$.

a) Integrate both sides of this equation to find a power series representation of $\tan^{-1} x$ (you'll need to determine the value of the constant that appears when you integrate: evaluate at $x = 0$ to do this).

b) We know that your series converges when $|x| < 1$. Show that it also converges for $x = 1$.

c) Use your power series the fact that $\pi = 4 \tan^{-1}(1)$ to find a series whose sum is π.